Graz University of Technology

Institut für Angewandte Mathematik

Univ.–Prof. Dr. Olaf Steinbach
Institutsvorstand

Steyrergasse 30
A 8010 Graz
Austria

Tel. +43(0)316 873-8120
Fax +43(0)316 873-8621

o.steinbach@tugraz.at
www.numerik.math.tugraz.at

DVR: 008 1833 UID: ATU 574 77 929

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Miniworkshop

Analysis and Numerics of Optimal Control Problems

Mittwoch, 19.10.2022, Steyrergasse 30, SR AE02

14.00–14.45	Fredi Tröltzsch (TU Berlin) On elliptic optimal control problems with control appearing nonlinearly in the state equation
14.45–15.30	Olaf Steinbach (TU Graz) (Space-time) finite element methods for optimal control problems with energy regularization
16.00–16.45	Richard Löscher (TU Graz) Numerical illustration of an abstract setting for distributed optimal control problems
16.45–17.30	Ulrich Langer (JKU Linz) Efficient iterative solvers for finite element optimality systems

On elliptic optimal control problems with control appearing nonlinearly in the state equation

Fredi Tröltzsch (Technische Universität Berlin)

An optimal control problem for a semilinear elliptic equation is discussed, where the control appears nonlinearly in the state equation but is not included in the objective functional. The existence of optimal controls is proved by a measurable selection technique. First-order necessary optimality conditions are derived and two types of second-order sufficient optimality conditions are established. A first theorem invokes a well-known assumption on the set of zeros of the switching function. A second relies on coercivity of the second derivative of the reduced objective functional. The results are applied to the convergence of optimal state functions for a finite element discretizion of the control problem.

The talk is on joint work with Eduardo Casas (Santander).

(Space-time) finite element methods for optimal control problems with energy regulatrization

Olaf Steinbach (Technische Universität Graz)

In this talk we present an abstract framework for the minimization of tracking type functionals subject to the solution of partial differential equations which defines an isomorphism between the control and state spaces. We present a priori error estimates for both the regularization error, and the discretization error, which results in an optimal choice of the regularization parameter with respect to the finite element mesh size. This theory also covers the case of state or control constraints. Applications include distributed optimal control problems subject to the Poisson equation, the heat equation, and the wave equation, and Dirichlet boundary control problems. We can also consider the control in L^2, but then the state space has to be defined accordingly.

The talk is based on joint work with U. Langer, R. Löscher, F. Tröltzsch, and H. Yang.

Numerical illustration of an abstract setting for distributed optimal control problems

Richard Löscher (Technische Universität Graz)

Building up on the abstract framework for distributed optimal control problems, in this talk selected examples are discussed and numerically analysed. First, we take a look at the Laplace equation and investigate the difference between the energy regularization and the, more standard, L^2-regularization. We further propose an adaptive refinement scheme, which will be complemented by an adaptive choice of the relaxation parameter. Moreover, constraints are discussed.

Second, we will show how the optimal control problem subject to the wave equation fits into the abstract setting. We will again consider the energy and L^2-regularization and compare the results to the elliptic case.

This talk is based on joint work with U. Langer, O. Steinbach and H. Yang.

Efficient iterative solvers for finite element optimality systems

Ulrich Langer (Johannes Kepler Universität Linz)

We consider iterative solvers for systems of algebraic equations resulting from the finite element discretization of reduced optimality systems characterizing the solution of elliptic, parabolic, and hyperbolic optimal control problems with L_2- and energy regularization. It turns out that, in the case of the L_2-regularization, the regularization parameter ϱ should behave like h^4 in the case of quasi-uniform meshes in order to get optimal L_2-norm error estimates between the computed finite element state and the traced desired state. With this choice, one can construct optimal preconditioners based on simple diagonal approximation of the mass matrix. These results remain true in the case of adaptively refined meshes when one uses variable regularization that can change from element to element. Similar results can be obtained for elliptic optimal control problems with energy regularization provided that ϱ behaves like h^2 in the case of quasi-uniform meshes. Finally, we will discuss some open problems, in particular, in connection with space-time solvers for the finite element optimality system arising from parabolic and hyperbolic optimal control problems

The talk is based on joint papers and ongoing work with Richard Löscher, Andreas Schafelner, Olaf Steinbach, Fredi Tröltzsch, and Huidong Yang.