
24. Workshop on
**Fast Boundary Element Methods and
Space-Time Discretization Methods**

Söllerhaus, 24.–27.9.2026

U. Langer, M. Schanz, O. Steinbach, W. L. Wendland (eds.)

**Berichte aus dem
Institut für Angewandte Mathematik**

Technische Universität Graz

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Program

Thursday, September 24, 2026	
15.00	Coffee
16.30–16.45	Opening
16.45–17.15	E. P. Stephan (Hannover) A least-squares FEM-BEM coupling method for the eddy current problem
17.15–17.45	M. Hartmann (Kiel) Numerical developments for modelling 3D ship hydrodynamics in a time-domain boundary element method
17.45–18.15	H. Gimperlein (Innsbruck) Boundary elements and model order reduction for sound emission problems
18.30	Dinner
Friday, September 25, 2026	
8.00–9.00	Breakfast
9.00–9.30	B. Gräßle (Zürich) Kernel-free BEM for general elliptic operators
9.30–10.00	N. Arora (Graz) A time-domain kernel independent fast multipole method based on strong recursive skeletonization using deterministic and randomized interpolative decomposition
10.00–10.30	T. Kramer (Graz) A numerical integration strategy for a time-domain higher-order isogeometric boundary element method
10.30–11.00	Break
11.00–11.30	C. Jerez-Hanckes (Stockholm) One Calderón operator to rule them all: Robust preconditioning under shape uncertainty equations
11.30–12.00	Q. H. Nguyen (Ghent) GMRES convergence for preconditioned weakly T-coercive equations and applications to electromagnetics
12.00–12.30	Z. Machaczek (Ostrava) Operator preconditioning in higher order BEM for 2D problems
12.30	Lunch
15.00	Coffee
15.30–16.00	A. Freiszlinger (Wien) Optimal convergence of adaptive BEM driven by functional error estimates
16.00–16.30	F. Nowakowsky (Twente) Adaptive space-time BEM for the heat equation with Neumann boundary conditions
16.30–17.00	C. Schwarz (Bayreuth) Collocation for the acceleration of the boundary element method for linear elasticity
17.00–17.15	Break
17.15–17.45	G. Gantner (Twente) Boundary elements for clamped Kirchhoff–Love plates
17.45–18.15	L. Swoboda (Gothenburg) A hybridizable discontinuous Galerkin method for wave propagation in elastic beam networks
18.30	Dinner

Saturday, September 26, 2026	
8.00–9.00	Breakfast
9.00–9.30	C. Urzua–Torres (Delft) Exterior square root
9.30–10.00	M. Reichelt (Hamburg) Elliptic space-time methods for time-periodic parabolic PDEs
10.00–10.30	T. Kaltenbacher (Graz) A space-time finite element method for the Stokes system
10.30–11.00	Break
11.00–11.30	C. Lin (Zürich) New integral representation of Green’s function for an acoustic impedance half-space
11.30–12.00	J. Lozano Simón (Bonn) Multilevel Monte Carlo methods for random nonlinear eigenvalue problems
12.00–12.30	D. Lukáš (Ostrava) EasyBEM - An open source component library
12.30	Lunch
13.30–18.00	Hiking Tour
18.30	Dinner
Sunday, September 27, 2026	
8.00–9.00	Breakfast
9.00–9.30	T. Roubal (Prague) Subdivision approximations of projected feasible regions in AC power flow
9.30–10.00	X. Claeys (Paris) Coarse space preconditioning for generalised optimized Schwarz methods
10.00–10.30	H. Egger (Linz) A coercive space-time variational approach to fractional diffusion problems
10.30	Closing

IABEM Symposium 2027

UCL, London, 20.–22.7.2027

https://tbetcke.github.io/iabem_london/

40. Chemnitz FEM Symposium (on tour)

TU Graz, 8.–10.9.2027

25. Söllerhaus Workshop on

Fast Boundary Element Methods and Space-Time Discretization Methods

30.9.–3.10.2027

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A time-domain kernel independent fast multipole method based on strong recursive skeletonization using deterministic and randomized interpolative decomposition

N. Arora, M. Schanz

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Chebyshev Interpolation based fast multipole method (ChebyshevFMM) approximate far-field interactions through a tensor-product representation requiring p^3 interpolation points in three dimensions for order p , and are particularly effective for low frequency kernels [1,2]. For high frequency oscillatory kernels, such as Helmholtz kernel, far-field interactions are no longer low rank, and their numerical rank increases with wavenumber and cluster size, necessitating higher interpolation order to maintain a prescribed accuracy [3,4]. This is particularly relevant in convolution quadrature boundary element formulations, where the integral equations are solved at a sequence of complex Laplace frequencies, for which a single interpolation order is possibly insufficient. In contrast, recursive skeletonization based fast multipole method (SkelFMM) approximates far-field interactions, within prescribed tolerance, by an equivalent set of skeleton points selected through interpolative decomposition (ID) to reproduce the effect of original sources in the far-field [5]. In this work, ChebyshevFMM and SkelFMM are investigated within Runge-Kutta convolution quadrature (CQ) boundary element framework for three dimensional transient acoustics, with particular emphasis on their low rank approximation behavior across CQ generated complex frequency spectrum, computational time, and storage requirements. Furthermore, deterministic and randomized ID within SkelFMM framework are compared to assess the efficiency of the skeletonization process. Numerical results demonstrate lower computational time and storage requirements for SkelFMM than for ChebyshevFMM.

References

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Coarse space preconditioning for Generalized Optimized Schwarz Methods

Xavier Claeys

UMA - ENSTA Paris, France

The Generalized Optimized Schwarz Method (GOSM) is a variant of Deprés algorithm, a domain decomposition strategy for the solution of harmonic wave propagation problems. It imposes transmission conditions through interfaces by means of a non-local exchange operator. Conducting our analysis at the continuous level, in an infinite dimensional setting, we propose a coarse space construction for the preconditioning of the GOSM formulation, and provide estimates for the convergence of GMRes applied to the preconditioned equation.

A coercive space-time variational approach to fractional diffusion problems

Herbert Egger

Institut für Numerische Mathematik, Johannes Kepler Universität Linz, Austria

We consider a fractional diffusion problem with temporal nonlocality acting on the diffusive flux. A coercive space–time variational formulation in Bochner-valued fractional Sobolev spaces is derived and the existence, uniqueness, and regularity of solutions are established. We further develop a conforming tensor-product Galerkin discretization and prove quasi-optimal error estimates in the anisotropic energy norm and improved convergence rates in weaker norms using duality arguments. In contrast to some space-time formulations for classical diffusion, the method preserves the causal structure of the evolution problem and leads to a time-stepping procedure with memory terms. On uniform time grids, the discrete history operator has a lower-triangular Toeplitz structure which enables an efficient implementation using fast recursive convolution techniques

Optimal convergence of adaptive BEM driven by functional-type error estimators

Alexander Freiszlinger

Institut für Analysis und Scientific Computing, TU Wien, Austria

We derive upper bounds of functional type for the potential error arising from boundary element discretizations of the Laplace–Dirichlet problem. These bounds are based on local auxiliary problems on vertex patches intersecting the boundary and the resulting a posteriori error estimator is shown to provide essentially constant-free upper bounds independent of the discretization. Moreover, in case of Galerkin BEM, the functional error estimator is locally equivalent to the well-studied residual error estimator. This equivalence result allows us to prove R -linear convergence of the functional error estimator and, together with a suitable mesh-refining strategy, to establish that the potential error as well as the functional error estimator converge with optimal rates with respect to the number of boundary elements. To illustrate the theoretical findings, we present a series of numerical experiments.

Boundary elements for clamped Kirchhoff–Love plates

Thomas Führer¹, Gregor Gantner², Norbert Heuer¹

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Building in particular on the analysis of [1, 2, 3], we present a Galerkin boundary element method for clamped Kirchhoff–Love plates with piecewise smooth boundary [4]. It is a direct method based on the representation formula and requires the inversion of the single-layer operator and an application of the double-layer operator to the Dirichlet data. We present trace approximation spaces of arbitrary order, required for both the Dirichlet data and the unknown Neumann trace. Our boundary element method is quasi-optimal with respect to the natural trace norm and achieves optimal convergence order under minimal regularity assumptions. Numerical experiments for smooth and non-smooth domains confirm predicted convergence rates.

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Boundary elements and model order reduction for sound emission problems

Heiko Gimperlein

Universität Innsbruck, Austria

We investigate a stable space-time formulation and model order reduction for long-time industrial sound emission problems, using boundary elements. To this end, we use a Galerkin formulation in space and time of the acoustic wave equation in 3d involving the hypersingular boundary integral operator. Our numerical experiments confirm that the resulting time stepping scheme is stable and accurate for acoustic problems in complex geometries, in contrast to alternative well-known schemes.

We then consider model order reduction techniques for the spatial discretization, based on the Fourier–Laplace transform into the frequency domain. In particular, based on work by Henriquez and Hesthaven, in an offline stage we compute accurate solutions of the frequency-domain problem at sample frequencies and extract a reduced basis of the space discretization using a Proper Orthogonal Decomposition. In an online phase, we solve the time dependent problem in this basis. The proposed methods are shown to be efficient for real-world problems, and we obtain very good agreement with physical acoustic measurements.

This lecture is based on joint works with B. Graf, C. Özdemir, S. Schneider, K. Urban, C. Guardasoni, and F. Henriquez.

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Kernel-free BEM for general elliptic operators

Benedikt Gräßle

Universität Zürich, Switzerland

Boundary integral formulations reduce the dimension of elliptic boundary value problems, but classical methods rely on explicit fundamental solutions and the evaluation of singular kernels—tools that are generally unavailable for variable coefficients. This talk investigates a kernel-free boundary element framework that replaces analytic kernels by a computable algebraic approximation of the boundary operator.

That approximation is obtained in a one-time preprocessing step from a volume Galerkin discretisation of the associated potential and then coupled with boundary discretisations to produce a boundary-only algebraic system. The method preserves dimension reduction and extends quasi-optimal boundary element discretisations beyond explicit Green's functions. Stability and quasi-optimal error bounds relate the boundary accuracy to the volume approximation. The structure of the resulting boundary operator is compatible with data-sparse compression techniques and is thus suited for efficient large-scale computations.

This talk is on joint work with Stefan A. Sauter (Uni Zürich).

One Calderón operator to rule them all: Robust preconditioning under shape uncertainty

Carlos Jerez Hanckes

KTH Stockholm, Sweden

Many-query electromagnetic scattering problems arising in uncertainty quantification, design, and inverse problems require the repeated solution of boundary integral equations on varying geometries. For the electric field integral equation (EFIE), Calderón operator preconditioning provides a robust discretization strategy, but constructing the geometry-dependent preconditioner can itself constitute a substantial fraction of the computational cost. This motivates a simple question: can one assemble a Calderón preconditioner once and reuse it across an entire family of geometries?

I will show that, for sufficiently controlled shape perturbations, a Calderón preconditioner constructed on a nominal geometry can indeed be reused while retaining robustness with respect to both the geometric parameters and spatial mesh refinement. The analysis combines bi-parametric operator-preconditioning theory with the shape-parametric regularity of the pulled-back Maxwell EFIE; recent holomorphy results provide the operator-norm perturbation estimates underlying the reuse argument.

Numerical experiments for smooth and nonsmooth geometric perturbations demonstrate that the frozen preconditioner closely tracks freshly assembled Calderón preconditioners while substantially reducing setup costs. I will also discuss the extension to stochastic Galerkin discretizations, where the same nominal preconditioner lifts naturally to the coupled stochastic system and remains robust under polynomial-chaos refinement. Finally, I will comment on the limits of reuse and on extensions toward wider parameter ranges and many-query electromagnetic computations.

Numerical developments for modelling 3D ship hydrodynamics in a time-domain boundary element method

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¹Institute of Maritime Technologies and Propulsion Systems, DLR, Geesthacht, Germany

²Elite Graduate Program Scientific Computing, University of Bayreuth, Germany

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The prediction of hydrodynamic loads on ships in waves and induced ship motions is of high relevance for the maritime industry. For example, the knowledge of the environmental loads could be used to optimize the energy consumption of waterborne vessels, and the short-term forecast of motion based on deterministic wave prediction to increase safety during offshore operations. Highly accurate and efficient physics-based numerical methods are key tools to provide such information to maritime stakeholders. Boundary element methods are predestined for this task and fill the gap between high-fidelity CFD methods such as RANSE or LES solvers, and low-order approaches such as semi-empirical models. In order to meet the requirements of accuracy and computational efficiency in the context of nonlinear wave-ship interaction, a high-order time-domain boundary element method is developed.

This contribution focuses on the extension of a boundary element method to three dimensions. It includes the description of techniques for 3D-geometry modelling and the spline-based reconstruction of surface data. Regarding to geometry modelling, an algorithmic pipeline using an open-source CAD kernel is developed to accurately evaluate the intersection curve of water and ship surface. This waterline calculation aims to capture the submerged part of the ship accurately, which results in more reliable motion predictions compared to those relying on the usual assumption of ship intersecting a flat surface. The reconstruction technique is relevant for time-domain simulations where the ship's change in position leads to the inclusion of previously hidden grid points. These data gaps in the proximity of the surface intersection are resolved by approximating the function values from the surrounding surface data. In addition to these two topics, current implementations for the extension to three-dimensions are outlined, and future tasks are mapped.

A space-time finite element method for the Stokes system

Tobias Kaltenbacher, Olaf Steinbach

Institut für Angewandte Mathematik, TU Graz, Austria

In this talk, we consider a space-time finite element method for the time-dependent Stokes system. While classical approaches rely on time-stepping schemes, we propose a fully space-time variational formulation in the Bochner setting. We first study the formulation on tensor-product space-time meshes and then extend it to completely unstructured meshes, allowing for a unified treatment of spatial and temporal discretization. Numerical experiments on arbitrary space-time meshes illustrate the flexibility and effectiveness of the proposed approach.

A numerical integration strategy for a time-domain higher-order isogeometric boundary element method

Thomas Kramer, Benjamin Marussig, Martin Schanz
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Higher-order methods offer rapid convergence and high accuracy. The small discretization errors provided by higher-order basis functions specifically require accurate computation of the matrix entries by numerical integration. We present a numerical integration scheme for a time-domain Boundary Element Method to solve acoustic scattering problems. Convolution quadrature based on multistage Runge-Kutta schemes translates the time-domain formulation into a sequence of elliptic boundary-value problems in the Laplace domain. These are discretized using isogeometric analysis yielding a higher-order method in both, space and time [1]. In this setting, the integrands are modified Helmholtz kernels that exhibit oscillatory behaviour and singularities. The proposed integration strategy enhances robustness and efficiency by addressing the two principal challenges: highly oscillatory behavior is handled using the method of steepest descents (via contour deformation) [2], while kernel (quasi-)singularities are treated by singularity subtraction [3] and the Duffy transformation. We apply a quasi-interpolation-based quadrature [3] that delivers a user-specified accuracy for both regular and regularized kernel evaluations. Numerical experiments demonstrate that the scheme maintains the accuracy required by higher-order spatial discretizations while reducing computational cost.

References

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New integral representation of Green's function for an acoustic impedance half-space

Chuhe Lin

Universität Zürich, Switzerland

We consider acoustic scattering in an upper half-space with an impedance boundary condition. The unbounded geometry motivates a boundary integral formulation based on the corresponding half-space Green's function. We present a new representation of the Green's function that separates the free-space, reflected, and impedance contributions, with the latter expressed through a slowly varying integral suitable for numerical evaluation. Based on this representation, we formulate a boundary integral equation for the unknown Neumann data and develop an efficient numerical approximation scheme. Numerical experiments demonstrate that the proposed approach accurately resolves highly oscillatory boundary data and exhibits the expected convergence behavior.

EasyBEM - An Open-Source Component Library

Dalibor Lukáš, Zbyšek Machaczek

TU VSB Ostrava, Czech Republic

We present a new open-source boundary element library. It relies on the Erichsen–Sauter–Schwab quadrature routines for the evaluation of single-layer, double-layer, and hypersingular boundary integral operators discretized by the Galerkin method. The evaluation can be offloaded to a GPU and performed only when the operator is applied to a vector. This is known as a matrix-free implementation. Together with a novel matrix-free operator preconditioner, we believe this approach provides one of the most efficient implementations currently available.

At present, the library supports 3D Laplace and elasticity problems. It can be easily used in both C++ and Python projects, and for Python we provide several examples to facilitate its use.

The library is available on www.github.com/easybem under the MIT licence.

Operator preconditioning in higher order BEM for 2D problems

Zbyšek Macháček

TU VSB Ostrava, Czech Republic

This talk deals with preconditioning the single layer boundary integral operator (BIO) using the hypersingular BIO. It is well-known that in order to obtain a system with a uniformly bounded condition number, a discontinuous-continuous pair of spaces with matching dimensions that satisfy a LBB condition is required. In our previous work we have considered the piecewise constant single layer for 2D and 3D problems, and proposed optimal preconditioners that utilize a piecewise polynomial hypersingular. We will present a generalization of this scheme that allows us to precondition piecewise polynomial discontinuous discretizations of the single layer BIO. Discretization spaces used for the preconditioner are once again piecewise polynomial. Both spaces used are defined on the same mesh. Numerical experiments will involve the Dirichlet BVP for the Laplace equation in 2 spatial dimensions.

GMRES convergence for preconditioned weakly T -coercive equations and applications to electromagnetics

Q. H. Nguyen, V. C. Le, and K. Cools

IDLab, Department of Information Technology, Ghent University–imec, Belgium

The convergence of preconditioned GMRES for weakly T -coercive operator equations and their Galerkin discretizations is established under appropriate assumptions. This class of equations encompasses several Maxwell problems, including the Maxwell source problem, the electric field integral equation (EFIE), and the Costabel FEM–BEM coupling formulation. The theory and its underlying assumptions are then specialized to the operator preconditioning framework [1]. The resulting analysis hence complements [1], since a uniformly bounded spectral condition number alone is known insufficient to guarantee mesh-independent GMRES convergence.

As an example, we apply operator preconditioning to a Galerkin discretization of the EFIE on a Lipschitz surface and show that all the required assumptions are naturally satisfied. In particular, the GMRES residual norms converge R -linearly and, for smooth surfaces, R -superlinearly. These results provide a theoretical explanation for the observed effectiveness of operator preconditioning and, in this case, of the classical Calderón preconditioner [2].

References

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Adaptive space-time BEM for the heat equation with Neumann boundary conditions

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²Universität Basel, Switzerland

We consider the space-time boundary element method (BEM) for the heat equation with prescribed initial and Neumann data [1, 2]. The method is based on a coercive boundary integral formulation on the lateral space-time boundary $\Sigma = (0, T) \times \partial\Omega$ involving the hypersingular operator, which is an isomorphism between the anisotropic Sobolev space $H^{1/2, 1/4}(\Sigma)$ and its dual. We propose a residual-type *a posteriori* error estimator that is an upper bound for the unknown BEM error on locally refined prismatic meshes, whose elements are tensor products $J \times K$ of elements in time J and space K . Under standard shape-regularity and local quasi-uniformity assumptions, and assuming, as for the Dirichlet problem [3], local parabolic scaling $|J| \simeq \text{diam}(K)^2$ on each prism $J \times K$, we prove that the energy error is controlled by the proposed estimator. This local estimator can be used to guide adaptive refinement, resulting in improved convergence rates compared to uniform refinement, as confirmed by our numerical experiments on two-dimensional spatial domains Ω .

References

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Elliptic space-time methods for time-periodic parabolic PDEs

Michael Reichelt

Helmut Schmidt University, Hamburg, Germany

For parabolic evolution problems equipped with initial and boundary conditions, it is meanwhile well known, that Hilbert type transformations yield elliptic variational formulations in space-time [1,2]. Recently, it has been shown, that these transformations arise naturally from the functional analytic concept of the *Polar Decomposition* [3]. This talk applies the concept to the surrogate ODE

$$\partial_t u(t) + \mu u(t) = f(t), \quad t \in (0, T), \quad u(0) = u(T),$$

deriving suitable transformation operators for the time-periodic case. We later generalize the approach to time-periodic parabolic PDEs, replacing $\mu > 0$ by an elliptic operator A in space. The resulting variational formulation is then discretized by a space-time finite element method, which is shown to be elliptic and quasi-optimal. Finally, we present linear as well as nonlinear numerical examples, demonstrating the applicability of the approach and comment on efficient solvers and FFT realizations of the transformation operators [4].

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Subdivision approximations of projected feasible regions in AC power flow

Tomas Roubal

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We consider the approximation of projected feasible sets

$$Y = \Phi(\Omega) \subset \mathbb{R}^m,$$

where Ω is defined by nonlinear equality and inequality constraints. We study a subdivision scheme based on successive box refinements and projection-based selection, together with a perturbed variant allowing inexact numerical tests. Two realizations are discussed, using either projection of box centers or the distance of $\Phi(x)$ from a box. The framework is applied to AC power flow feasibility problems, both with fixed PV/PQ bus types and with PV/PQ switching, and is illustrated on MATPOWER case30.

Collocation for the acceleration of the boundary element method for linear elasticity

Christina Schwarz

Chair of Scientific Computing, University of Bayreuth, Germany

Solving the mixed boundary value problem from linear elasticity by the boundary element method generally leads to non-sparse matrices. To reduce memory requirements, we approximate the arising operators using hierarchical matrices [1].

Furthermore, the Galerkin discretization is replaced by collocation in order to accelerate the simulations, employing a mixed approximation [2] to avoid the hypersingular operator. In addition, the computational efficiency of the inner-outer iteration scheme used to solve the Schur complement system is improved by inexact GMRes and inexact matrix-vector products [3, 4]. Finally, the adaptive cross approximation for the construction of the hierarchical matrices is accelerated by interpolation, exploiting the existence of two nested grids which are required for stability.

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Multilevel Monte Carlo methods for random nonlinear eigenvalue problems

Jürgen Dölz, Jimena Lozano Simón

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We develop a multilevel Monte Carlo (MLMC) estimator to compute statistical quantities of interest of solutions to parameter-dependent nonlinear eigenvalue problems. The main difficulty for this problem is that parametrized eigenpairs often lack a globally well-defined parameter-to-solution map due to crossings and bifurcations. These phenomena render eigenvalue-tracking infeasible in multi-dimensional parameter domains. To overcome this issue, we propose both an Eulerian and a Lagrangian perspective on parameter-to-solution maps for nonlinear eigenvalue problems, which remain well-defined even in the presence of local crossings and bifurcations. Their implementation is based on a straightforward modification of Beyn’s contour integral approach. Building on these maps, we propose an MLMC estimator that does not rely on matching eigenvalues or eigenvectors across samples or discretization levels. Nested approximation spaces and inter-level prolongation are only required when statistical quantities of interest related to the eigenspaces are of interest, but not for quantities of interest related to the eigenvalues. We demonstrate the feasibility and validity of our approach by numerical examples using a quasi-Monte Carlo method, including a nonlinear formulation of the Laplace Dirichlet eigenvalue problem on random domains in three dimensions motivated by the work of Steinbach and Unger. To the best of our knowledge, this is the first work considering the approximation of statistical quantities of interest for random nonlinear eigenvalue problems.

A Least Squares FEM-BEM coupling method for the eddy current problem

Ernst P. Stephan

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We analyze a least squares formulation for the fem-bem solution of the eddy current problem. In the bounded interior domain D_c Maxwell's equations are formulated as a first order system, whereas in the exterior domain D the harmonic function is represented via the Stratton-Chu formula. Extending the approach in [1] for second order elliptic problems, the transmission problem is converted into a quadratic minimization problem over $X := L^2(D_c) \times H(\text{curl}; D) \times H^{-1/2}(\text{div}_\Gamma; \Gamma)$. This functional is constructed via the inner products of the dual spaces $L^2(D_c) \times H^{-1}(\text{div}; D) \times H^{-1/2}(\text{curl}_\Gamma; \Gamma)$. For the discretization of X we are using Nedelec elements and Raviart-Thomas elements and brick elements. The discrete solution is constructed by representing the above inner products via appropriate preconditioners such as multigrid.

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A hybridizable discontinuous Galerkin method for wave propagation in elastic beam networks

Lucia Swoboda

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Network models are used to describe complex structures found in fiber-based materials such as paper, or biological tissues. In these models, fibers are represented as edges connected at nodes, and are modeled as Timoshenko beams to capture the mechanical behavior of the material. Understanding wave propagation in such networks is important for predicting material response and improving performance in applications such as papermaking.

We present a hybridizable discontinuous Galerkin (HDG) method for the spatial discretization of wave propagation on fiber networks, combined with a Crank–Nicolson scheme for time integration. Through hybridization, the problem is reformulated as a symmetric positive definite system on the network nodes. The HDG spatial discretization achieves arbitrary-order convergence under mesh refinement without increasing the size of the global system. We establish convergence and error estimates, supported by numerical experiments.

Exterior square root

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When designing space-time methods for Maxwell's equations, it is important to respect their rich geometric structure, which has important implications for the performance of numerical methods. A classical example in computational electromagnetics is the use of suitable vectorial finite element basis functions for the spatial discretization of Maxwell's equations [4].

To identify the geometric structures that should be preserved after discretization, we formulate Maxwell's equations in terms of differential forms on four-dimensional spacetime. Let $\mathcal{F}$ denote the electromagnetic field, $\mathcal{G}$ the electromagnetic excitation, and $\mathcal{J}$ the electric charge-current. Then Maxwell's equations read $d\mathcal{F} = 0$ and $d\mathcal{G} = \mathcal{J}$.

The geometry of spacetime enters through the constitutive relation connecting the electromagnetic field and the electromagnetic excitation. Assuming locality and linearity, it is described by a constitutive operator $\mathcal{M} : \Lambda^2 V \rightarrow \Lambda^2 V$ such that $\mathcal{G} = \mathcal{M}(\mathcal{F})$.

In the conventional formulation of Maxwell's equations, $\mathcal{M}$ is determined by the spacetime metric. In particular, the metric induces the constitutive relation through the Hodge operator. While the construction of $\mathcal{M}$ from a given metric is well understood, the converse question is more subtle: given a constitutive operator, under which conditions does it arise from a spacetime metric, and to what extent can that metric be recovered?

Previous work [3,5] made major contributions to this problem, but some assumptions remained coordinate dependent. In this talk, we address the converse question by studying the existence of an *exterior square root* associated with $\mathcal{M}$ [1,2], using differential forms to keep the formulation as coordinate-independent as possible.

This question is relevant for numerical methods because it clarifies the geometric structure underlying electrodynamics and thereby helps us understand how different (3+1)-dimensional representations of the same four-dimensional theory may lead to different discretization strategies.

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