Technische Universität Graz

Söllerhaus Workshop on

Domain Decomposition Methods

Söllerhaus, 13.10.–16.10.2011

A. Klawonn, O. Steinbach (eds.)

Berichte aus dem
Institut für Numerische Mathematik

Book of Abstracts 20011/5

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Program

Thursday, October 13, 2011	
15.00–16.20	Coffee
16.20–16.30	Opening
16.30–17.15	T. Dickopf (Lugano) Quantitative studies of transfer operators between non–nested finite element spaces
17.15–18.00	C. Pechstein (Linz) An abstract two–level additive Schwarz method for systems with high contrast coefficients
18.30	Dinner
Friday, October 14, 2011	
9.00–9.45	U. Rüde (Erlangen) Solving PDE with millions of cores
9.45–10.30	M. Neumüller (Graz) A hybrid DG method in space and time
10.30–11.00	Coffee
11.00–11.45	S. Gippert (Essen) FETI–DP for linear elasticity problems with compressible and almost incompressible material components in 3D
11.45–12.30	E. Karabelas (Graz) Modelling of the coupled electromechanical activation of the human heart
12.30	Lunch
16.00–16.30	Coffee
16.30–17.15	M. Jarosava (Ostrava) Hybrid total FETI
17.15–18.00	H. Yang (Linz) Two finite element methods on hybrid meshes and their applications on the fluid–structure interaction problems
18.30	Dinner

Saturday, October 15, 2011	
9.00–9.45	T. Kozubek (Ostrava) Stable computation of a generalized inverse in FETI methods
9.45–10.30	C. Augustin (Graz) Considerations about FETI for linear and nonlinear elasticity
10.30–11.00	Coffee
11.00–11.45	G. Of (Graz) Coupling methods for interior penalty discontinuous Galerkin finite element methods and boundary element methods
12.00	Lunch
13.00–18.00	Hiking Tour
18.30	Dinner
Sunday, October 16, 2011	
9.00–9.45	Z. Dostal (Ostrava) Scalable FETI based algorithms for transient contact problems: theory, implementation, and numerical experiments
9.45–10.30	O. Steinbach (Graz) Boundary element methods for variational inequalities
10.30	Closing and Coffee

Considerations about FETI for Linear and Nonlinear Elasticity

C. Augustin, O. Steinbach
TU Graz, Austria

In this talk we will rehash the main principles of the well known Finite Element Tearing and Interconnecting (FETI) Method. At the example of linear elasticity different local solution methods, as well as global preconditioning techniques are discussed. Furthermore we will compare the classical FETI approach with all-floating FETI.

Concerning nonlinear elasticity the focus will be on structural models for biological tissue, for example arterial walls or cardiac tissue. These materials are anisotropic due to a preferential orientation of collagen fibers and consist of several layers. The resulting nonlinear models lead to very complex and time-consuming algorithms. Hence parallel methods such as FETI are very well suited to treat such problems. In this case theoretical aspects are discussed and some numerical examples are given. Eventually, we will show perspectives and our future work within the field of biomechanics, e.g. coupling with fluid mechanics or electricity.

Quantitative studies of transfer operators between non–nested finite element spaces

T. Dickopf, R. Krause
University of Lugano, Switzerland

We present new quantitative studies of transfer operators between finite element spaces associated with unrelated meshes. Our extensive numerical studies in 3D provide substantial insight into non–nested information transfer with the first numerical comparison of different transfer operators between non–nested finite element spaces. We consider the standard finite element interpolation, Clément's quasi–interpolation with different local polynomial degrees, the global L^2–orthogonal projection, a local L^2–quasi–projection via a discrete inner product, and a pseudo–L^2–projection defined by a Petrov–Galerkin variational equation with a discontinuous test space. The studies do not only provide estimates of the H^1–stability constants but also allow for determining quantitative differences between the transfer operators. Understanding their qualitative and quantitative behaviors in this new level of detail is interesting per se; moreover, it is highly relevant in the context of discretization and solution techniques which make use of different non-nested meshes. It turns out that the pseudo-L^2–projection approximates the actual L^2–orthogonal projection best. In addition, the stability constants experimentally prove to be lower than or equal to one. Here, as a particular example, we apply our findings to the construction of multilevel methods based on a hierarchy of non–nested meshes.

Scalable FETI based algorithms for transient contact problems: theory, implementation, and numerical experiments

T. Brzobohatý, Z. Dostál, T. Kozubek, A. Markopoulos, V. Vondrák
VŠB–TU Ostrava, Czech Republic

We report the results of our research in development of the algorithms with asymptotically linear complexity for the solution of transient contact problems of elasticity that are capable to reduce the time of computation nearly proportionally to the number of available processors. Our talk covers 2D and 3D problems discretized by the finite element or boundary element method, possibly with "floating" bodies. A characteristic feature of the problems considered in our talk is a strong nonlinearity due to the interface conditions. Since even the algorithms for the solution of linear problems have at least the linear complexity, it follows that a scalable algorithm for contact problem has to treat the nonlinearity in a sense for free. After introducing the relations that describe the equilibrium of a system of elastic bodies in mutual contact, we briefly review the TFETI/TBETI (total finite/boundary element tearing and interconnecting) based domain decomposition methodology adapted to the solution of the transient contact problems. Recall that TFETI differs from the classical FETI or FETI2 as introduced by Farhat and Roux by imposing the prescribed displacements by the Lagrange multipliers and treating all subdomains as floating. The time discretization is carried out by the Newmark method. Then we present our in a sense optimal algorithms for the solution of the resulting quadratic programming problems. A unique feature of these algorithms is their capability to solve the class of such problems with homogeneous equality constraints and separable inequality constraints in O(1) matrix–vector multiplications provided the spectrum of the Hessian of the cost function is in a given positive interval. For longer time steps, we present the preconditioning by a conjugate projector that affects also the nonlinear steps. Finally we put together the above results to develop scalable algorithms for the solution of each time step. We illustrate the results by numerical experiments.

FETI–DP for linear elasticity problems with compressible and almost incompressible material components in 3D

S. Gippert, A. Klawonn, O. Rheinbach
Universität Duisburg–Essen, Germany

In this talk, a domain decomposition method for a class of elasticity problems with compressible and almost incompressible material components is considered. The problem is discretized with an inf–sup stable mixed finite element method with discontinuous pressure variables. The latter are statically condensated which leads to a symmetric positive definite linear system. As a domain decomposition method, a FETI–DP algorithm with primal vertices and primal edge averages is applied. The distribution of the material parameters is chosen such that in each subdomain there is an almost incompressible inclusion surrounded by a compressible hull. The condition number is found to depend only on the material parameters of the hull and also on its thickness. Numerical results confirming our theoretical findings are presented.

Hybrid Total FETI

M. Jarošová, M. Menšik, Alex Markopoulos
Centre of Excellence IT4Innovations, VŠB–TU Ostrava, Czech Republic

We propose a hybrid FETI method [1] based on our variant of the FETI type domain decomposition method called Total FETI [2]. Our hybrid method was developed in an effort to overcome the bottleneck of classical FETI methods, namely the bound on the dimension of the coarse space due to memory requirements.
We first decompose the domain into relatively large clusters that are completely separated, and then we decompose each cluster into smaller subdomains that are joined partly by Lagrange multipliers λ_0 in selected interface variables or in averages if the transformation of basis is applied. The continuity in the rest of interface variables and also the Dirichlet condition are enforced by Lagrange multipliers λ_1. This decomposition leads to the algorithm, where TFETI is used on two levels. The similar hybrid FETI method was considered in [4] and [3].
The results of numerical experiments on benchmark from the linear elasticity will conclude the talk.

References

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[3] A. Klawonn, O. Rheinbach, Highly scalable parallel domain decomposition methods with an application to biomechanics, ZAMM, 90, No. 1, pp.5-32, 2010.

[4] Jungho Lee: A hybrid domain decomposition method and its applications to contact problems in mechanical engineering. PhD thesis, New York University, 2009.

Modelling of the coupled electromechanical activation of the human heart

E. Karabelas, O. Steinbach
TU Graz, Austria

In this talk we shall focus on mathematical modelling aspects in bioelectromechanics of the myocardium. We shall first present the bidomain equations, modelling the electric activation of the heart. Secondly we will give the pure mechanical models. Afterwards we will show, how this two models may be coupled and present the resulting equations.
We shall also touch on issues of (unique) solvability and discuss some particular models in literature.

Stable computation of a generalized inverse in FETI methods

T. Kozubek

Centre of Excellence IT4Innovations, VŠB–TU Ostrava, Czech Republic

Effective implementation of some efficient FETI methods assumes application of a direct method for solving a system of linear equations with a symmetric positive semidefinite matrix $\mathbf{A}$. The latter usually comprises a triangular decomposition of a nonsingular diagonal block $\mathbf{A}_{\mathcal{J}\mathcal{J}}$ of the stiffness matrix $\mathbf{A}$ of a subdomain and an effective evaluation of the action of a generalized inverse of the corresponding Schur complement. The goal is to review our results which address both problems. We present a procedure which first identifies a well-conditioned positive definite diagonal block $\mathbf{A}_{\mathcal{J}\mathcal{J}}$ of $\mathbf{A}$, then decomposes $\mathbf{A}_{\mathcal{J}\mathcal{J}}$ by the Cholesky decomposition, and finally evaluates a generalized inverse of the Schur complement $\mathbf{S}$ of $\mathbf{A}_{\mathcal{J}\mathcal{J}}$. The Schur complement $\mathbf{S}$ is typically very small, so the generalized inverse can be effectively evaluated by the SVD. If the rank of $\mathbf{A}$ or a lower bound on the nonzero eigenvalues of $\mathbf{A}$ are known, then the SVD can be implemented without any "epsilon". Moreover, if the kernel of $\mathbf{A}$ is known, then it is shown that the SVD can be replaced by effective regularization of the Schur complement.
Finally, if the kernel of $\mathbf{A}$ is known, then the above approach can be replaced by effective and sparse regularization directly of the matrix $\mathbf{A}$. The inverse of the regularized $\mathbf{A}$ will be simultaneously a generalized inverse to $\mathbf{A}$. This justifies factorization of the regularized $\mathbf{A}$ using any standard Cholesky type decomposition for nonsingular matrices to evaluate the action of its generalized inverse on a vector. This ingredient significantly simplifies implementation of the Total FETI method.

References

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[3] Kucera, R., Kozubek, T., Markopoulos, A., Machalova, J.: On the Moore-Penrose inverse in solving saddle-point systems with singular diagonal blocks. Accepted for publishing in NLA, 2011.

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A hybrid DG method in space and time

M. Neumüller, O. Steinbach
TU Graz, Austria

For evolution equations we present a space-time method based on Discontinuous Galerkin finite elements. Space-time methods have advantages when we have to deal with moving domains. This method results in a large system of linear equations which are handled by a hybrid formulation which allows the use of parallel solution algorithms. Numerical examples for the heat equation and the Navier-Stokes equations will be given, which show the expected convergence of this approach.

Coupling methods for interior penalty discontinuous Galerkin finite element methods and boundary element methods

G. Of[1], G. Rodin[2], O. Steinbach[1], M. Taus[2]
[1]TU Graz, Austria
[2]The University of Texas at Austin, USA

In this talk we present three new coupling methods for interior penalty discontinuous Galerkin finite element methods and boundary element methods. The new methods allow one to use discontinuous basis functions on the interface between the subdomains represented by the finite element and boundary element methods. This feature is particularly important when discontinuous Galerkin finite element methods are used. Error and stability analysis is presented for some of the methods. Numerical examples suggest that all three methods exhibit very similar convergence properties, consistent with available theoretical results.

An abstract two–level additive Schwarz method for systems with high contrast coefficients

C. Pechstein

Johannes Kepler Universität Linz, Austria

Robust domain decomposition solvers for the finite element discretization of scalar elliptic problems with heterogeneous (high contrast, or multiscale) coefficients have been studied for some time. This includes both iterative substructuring methods and overlapping Schwarz methods. Using weighted Poincaré inequalities and/or spectral theory, robustness can theoretically be guaranteed in a large variety of cases. However, there is only little theory available for *systems* of PDEs, such as linearized elasticity or problems in $H(\mathrm{curl})$.
In this talk, we consider an abstract framework for overlapping Schwarz methods for variationally posed systems of PDEs. Essentially, we only require positive semi-definite element stiffness matrices and their connectivity. As usual, the local subspaces are defined on overlapping subdomains. The key ingredient is an abstract coarse space constructed from a particular eigenproblem in the overlap of each subdomain as well as an algebraic partition of unity. We provide a rigorous and robust convergence theory and show some numerical results for the cases of Darcy and linearized elasticity.

The talk is on joint work with V. Dolean (Université de Nice), P. Hauret (Michelin), F. Nataf (Universié Pierre et Marie Curie), R. Scheichl (University of Bath), and N. Spillane (Université Pierre et Marie Curie).

Solving PDE with millions of cores

U. Rüde
Friedrich–Alexander Universität Erlangen–Nürnberg, Germany

Technology has reached a stage, where individual processors can hardly be made faster. More computing power can be delivered only by more parallelism. Thus, multi-core processors are now ubiquitous, and this trend will even accelerate. Current projections indicate that by the end of the decade, high end supercomputers may employ hundreds of millions of processor cores. However, these architectures will have severe limitations and constraints, such as those dictated by the need to limit the energy consumption. In particular, memory access, communication, and synchronization will increasingly become computational bottlenecks since these are the most energy intensive operations inside a computing system. Therefore, exploiting such architectures will require that we rethink parallel computing. For many numerical computations, we may be forced to develop new innovative algorithms that permit more parallelism, that require less synchonization, that reduce communication, and which use more efficient memory access patterns. The talk will review the current technology trends and their alternatives and will then proceed to present some ideas and first experiments that may be helpful in the coming age of massively parallel numerical simulation.

Boundary element methods for variational inequalities

O. Steinbach
TU Graz, Austria

Variational inequalities to be considered in $H^{1/2}(\Gamma)$ result from different applications, e.g. boundary value problems with boundary conditions of Signorini type, contact problems in elasticity, or from constrained optimal control problems. In this talk we will discuss both the error analysis of the boundary element solution, and suitable iterative solution strategies.

Two finite element methods on hybrid meshes and their applications on the fluid-structure interaction problems

H. Yang
RICAM Linz, Austria

Two finite element methods on hybrid meshes containing tetrahedral, pyramidic, prismatic and hexahedral elements are considered. The first method is based on a conforming subdivision of pyramids, prisms and hexahedra into pure tetrahedra without introducing any new node, namely, in a pure geometrical way. The second one is constructed by a conforming subdivision of pyramids, prisms and hexahedra into pure tetrahedra with new nodes on the quadrilateral face centers and on the non-tetrahedral volumn centers, where each introduced additional nodal degree of freedom is eliminated by local averaging over those degrees of freedom associated to its original neighbouring nodes, namely, in an algebraic way. Both methods preserve the same number of nodal degrees of freedom as the number of original nodes of the hybrid mesh. The methods are used and compared with each other in the finite element discretization of the structure and fluid sub-problems which appear in partitioned algorithms for solving the fluid-structure interaction problems.

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Erschienene Preprints ab Nummer 2010/1

2010/1	G. Of, T. X. Phan, O. Steinbach: Boundary element methods for Dirichlet boundary control problems.
2010/2	P. D. Ledger, S. Zaglmayr: hp–Finite element simulation of three–dimensional eddy current problems on multiply connected domains.
2010/4	S. Engleder, O. Steinbach: Boundary integral formulations for the forward problem in magnetic induction tomography.
2010/5	Z. Andjelic, G. Of, O. Steinbach, P. Urthaler: Direct and indirect boundary element methods for magnetostatic field problems.
2010/8	A. Klawonn, U. Langer, L. F. Pavarino, O. Steinbach, O. B. Widlund (eds.): Workshop on Domain Decomposition Solvers for Heterogeneous Field Problems, Book of Abstracts.
2010/9	O. Steinbach, G. Unger: Convergence analysis of a Galerkin boundary element method for the Dirichlet Laplacian eigenvalue problem.
2010/10	O. Steinbach (ed.): Workshop on Computational Electromagnetics, Book of Abstracts.
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2011/1	O. Steinbach, G. Unger: Convergence orders of iterative methods for nonlinear eigenvalue problems.
2011/2	M. Neumüller, O. Steinbach: A flexible space–time discontinuous Galerkin method for parabolic initial boundary value problems.
2011/3	G. Of, G. J. Rodin, O. Steinbach, M. Taus: Coupling methods for interior penalty discontinuous Galerkin finite element methods and boundary element methods.
2011/4	U. Langer, O. Steinbach, W. L. Wendland (eds.): 9th Workshop on Fast Boundary Element Methods in Industrial Applications, Book of Abstracts.