Technische Universität Graz

22. Workshop on

Fast Boundary Element Methods and Space-Time Discretization Methods

Söllerhaus, 26.–29.9.2024

U. Langer, M. Schanz, O. Steinbach, W. L. Wendland (eds.)

Berichte aus dem Institut für Angewandte Mathematik

Book of Abstracts 2024/5

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Program

Thursday, September 26, 2024	
15.00	Coffee
16.15–16.30	Opening
16.30–17.00	Markus Bause (Hamburg) Space-time finite element methods for the Navier–Stokes system: Discretizations, solver, and analysis
17.00–17.30	Thomas Führer (Santiago) Well-posedness of first-order formulations of wave equations and discretization by space-time finite elements
17.30–18.00	Christian Köthe (Graz) Space-time least-squares FEM for convection-diffusion problems
18.00–18.30	Christina Schwarz (Bayreuth) A mixed approximation of the boundary element method in linear elasticity
18.30	Dinner
Friday, September 27, 2024	
8.00–9.00	Breakfast
9.00–9.30	Gregor Gantner (Bonn) Space-time FEM-BEM couplings for parabolic transmission problems
9.30–10.00	Günther Of (Graz) A non-symmetric space-time coupling of finite and boundary element methods for a parabolic-elliptic interface problem
10.00–10.30	Matteo Ferrari (Wien) Is the one-equation coupling of finite and boundary element methods always stable? In the continuous setting, yes!
10.30–11.00	Break
11.00–11.30	Martin Averseng (Angers) A substructuring preconditioner for the Laplace hypersingular integral equation on multi-screens
11.30–12.00	Ignacia Fierro–Piccardo (London) Spectral properties of the OSRC-preconditioned EFIE
12.00–12.30	Zbyšek Machaczek (Ostrava) Operator preconditioning in boundary element methods avoiding dual meshes
12.30	Lunch
14.00–14.30	Wolfgang Wendland (Stuttgart) On the construction of the Stokes flow in a domain with cylindrical ends
14.30–15.00	Ernst Stephan (Hannover) Higher order non-conforming FE/BE coupling for 2-dim. eddy current problems
15.00–15.30	Moritz Hartmann (Geesthacht) Direct, time-domain boundary element method in Galerkin formulation for the analysis of coupled wave-body dynamics in hydrodynamic applications
15.30–16.00	Coffee
16.00–16.30	Merle Backmeyer (Darmstadt) IgANets in H(curl) and its trace spaces
16.30–17.00	Michael Reichelt (Graz) Finite element geometric calculus for elliptic problems
17.00–17.15	Break
17.15–17.45	Remo von Rickenbach (Basel) Anisotropic wavelet matrix compression of integral operators
17.45–18.15	Richard Löscher (Graz) A space-time reduced basis method for the wave equation
18.30	Dinner

Saturday, September 28, 2024	
8.00–9.00	Breakfast
9.00–9.30	Xavier Claeyes (Paris) Boundary integral formulation for acoustic scattering in fractal geometries
9.30–10.00	Michael Multerer (Lugano) p-multilevel Monte Carlo for acoustic scattering from large deviation rough random surfaces
10.00–10.30	Anouk Wisse (Delft) Convergence of Calderón residuals
10.30–11.00	Break
11.00–11.30	Marco Zank (Wien) Space-time BEM for the wave equation for flat objects
11.30–12.00	Timo Betcke (London) Component libraries for fast boundary element simulations
12.00–12.30	Ignacio Labarca–Figueroa (Innsbruck) Boundary element method for dilute colloidal suspensions under a shear flow
12.30	Lunch
16.30–17.00	Helmut Harbrecht (Basel) Wavelet compressed, modified Hilbert transform in the space-time discretization of the heat equation
17.00–17.30	Martin Schanz (Graz) 3D-ACA accelerated time domain boundary element method: FMM and H-matrix based approaches
17.30–18.00	Olaf Steinbach (Graz) Space-time finite element methods in thermoelasticity
18.30	Dinner
Sunday, September 29, 2024	
8.00–9.00	Breakfast

23. Söllerhaus Workshop on

Fast Boundary Element Methods and Space-Time Discretization Methods

24.9.–27.9.2025

A substructuring preconditioner for the Laplace hypersingular integral equation on multi-screens

Martin Averseng

Université d'Angers, France

We present a preconditioning method for the linear systems arising from the boundary element discretization of the Laplace hypersingular equation on a multiscreen in 3D. We analyze a substructuring domain-decomposition preconditioner. We prove that the condition number of the preconditioned linear system grows poly-logarithmically with H/h, the ratio of the coarse mesh and fine mesh size, and our numerical results indicate that this bound is sharp.

IgANets in H(curl) and its trace spaces

Merle Backmeyer[1,3], Stefan Kurz[1], Matthias Möller[2], Sebastian Schöps[3]

[1]Seminar for Applied Mathematics, ETH Zurich, Switzerland
[2]Delft Institute of Applied Mathematics, TU Delft, Netherlands
[3] Computational Electromagnetics Group, TU Darmstadt, Germany

Solving partial differential equations accurately and efficiently is crucial in computational science, especially for complex physical phenomena such as electromagnetic (EM) scattering. One established method to address this problem is through integral equations (IEs), specifically the electric field integral equation (EFIE). It requires precise geometric representation of the boundary. Isogeometric analysis (IGA) has emerged as a promising method that brings together computer-aided design (CAD) and finite element analysis (FEA) using spline-based functions for geometry and solutions. The spline spaces in [1] form a discrete de Rham sequence allowing for consistent field representations on volumes and surfaces. While robust and accurate, this method can be computationally intensive.
Physics-informed neural networks (PINNs) are another approach for solving PDEs that provide rapid post-training evaluations. PINNs incorporate physical laws in the loss function allowing the neural network to learn the solution at discrete collocation points. When extended to deep operator networks (DeepONets), they have the ability to learn operators, making them applicable to entire problem classes (e.g. with varying geometry) rather than just single problem setups. This capability makes DeepONets particularly attractive for optimization tasks. However, their heuristic nature and lack of a rigorous theory limit their reliability and pose challenges in ensuring that the solutions are physically meaningful.
Combining IGA's spline framework with DeepONets' computational speed, IgANets train neural networks to learn the coefficients of spline basis functions. This hybrid method has shown success for the volumetric discretizations of Poisson equation [2].
Our work adopts this approach for scattering and radiation field problems in the frequency domain represented by a surface IE. The network is trained on a 3D scattering/radiation problem, and its prediction accuracy is verified against an analytical solution. In a next step, training is extended for varying geometric configurations. Once trained, the network is applied to unseen geometries, demonstrating its generalization capability. This project is the first step in validating IgANets as an efficient numerical tool for solving EM problems.

References

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Space-time finite element methods for the Navier–Stokes system: Discretizations, solver and analysis

Markus Bause

Helmut Schmidt University Hamburg,
Faculty of Mechanical and Civil Engineering, Hamburg, Germany

Space-time finite element methods (STFEMs) feature the natural construction of higher order discretization schemes for partial differential equations and coupled systems. They offer the potential to inherit most of the rich structure of the continuous problem, while maintaining stability, and achieve accurate results on computationally feasible grids. The solution of the arising algebraic systems remains a challenging task.

Firstly, we present our STFEMs for simulating efficiently in three space dimensions solutions to the Navier–Stokes equations [3,4] and related problems [2]. Arbitrary order discontinuous Galerkin methods for the time discretization and inf-sup stable finite element pairs with discontinuous pressure for the space discretization are applied in a local (time stepping) approach. For solving the Newton linearized algebraic systems, GMRES iterations with Geometric Multigrid Preconditioning based on a local Vanka smoother are applied. The performance properties of the approach are illustrated for large scale benchmark problems.

Secondly, we address the optimal order approximation of the pressure trajectory for an equal-order in time discretization of the velocity and pressure [1]. For simplicity, piecewise linear polynomials and the Stokes system are considered. In the literature, the pressure approximation has attracted less attention than the one of the velocity, even though being of high importance for applications. This might be due to difficulties related to the processing of the initial pressure. By post-processing techniques using collocation conditions, an optimal order pressure trajectory is defined. Alternatively, interpolation is proposed. Error estimates are presented and their proofs are sketched.

References

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Component libraries for fast boundary element simulations

Timo Betcke
Department of Mathematics, University College London, UK

When designing boundary element codes, whether it is Nystrom or Galerkin implementations, many of the core library tasks are identical. One needs a surface grid library, direct evaluations of Green's functions, quadrature formulas, dense assembly and or fast assembly via accelerated methods. Even such simple tasks as a grid library or direct evaluation of Green's functions are, even though mathematically simple, computationally complex. For example, when evaluating Laplace kernels we want to be able to either assemble interaction matrices or evaluate kernel sums, support SIMD acceleration at least on x86 and ARM, and allow optional multithreading, all of which amounts to several thousand lines of code just for direct kernel evaluation. For grids we want to have arbitrary degree element types, basis functions, etc. Once we arrive at fast evaluation via FMM/H-Matrices the complexity grows significantly.
In this talk we discuss how we tackle this issue by splitting up our monolithic large boundary element codes into small component libraries that can be independently used and are easy to deploy in different projects. While written in Rust, we are depending on well defined C interfaces for inter-language communication and integration into scripting languages.
We are demonstrating a number of code samples and benchmarks for our various components, including evaluators, assembly, and FMM, and provide a glimpse into further developments that are currently ongoing to create a simple set of libraries for the design of boundary element methods.

Boundary integral formulation for acoustic scattering in fractal geometries

A. Caetano[1], S. N. Chandler–Wilde[2], X. Claeys[3], A. Gibbs[4], D. P. Hewett[4], A. Moiola[5]

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[2]Dept. Mathematics and Statistics, Univ. Reading, United Kingdom
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[4]Dept. Mathematics, University College London, United Kingdom [5]Dipt. Matematica "F. Casorati", Univ. Pavia, Italy

Considering the Helmholtz equation with Dirichlet boundary condition posed in the exterior of a highly irregular set, we focus on the case where the scatterer is a d-dimensional set with a potentially non-integer value of d. Besides a proper functional framework, we will discuss a boundary integral formulation for this problem, as well as mapping properties and well posedness of the integral operators and connections with other existing approaches that cover scattering by irregular potentially non-Lipschitz objects. We shall conclude by describing a concrete strategy for the numerical treatment of such formulations, and show actual numerical results.

References

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Is the one-equation coupling of finite and boundary element methods always stable? In the continuous setting, yes!

Matteo Ferrari

Fakultät für Mathematik, Universität Wien, Austria

We consider the non-symmetric coupling of finite and boundary elements to solve second order uniform elliptic partial differential equations defined in unbounded domains. Numerical tests suggest stability for all Lipschitz interfaces and elliptic diffusion matrices. However, the ellipticity of the associated bilinear form is guaranteed if and only if a precise (and sharp) condition is satisfied, that relates the minimal eigenvalue of the diffusion matrix to the contraction constant of the shifted double-layer integral operator. Through a T-coercivity argument, we show that stability can be proved in standard Sobolev spaces even if the above relation does not hold.
In this talk, we discuss the proof of this result and we highlight what is missing in order to apply this stability analysis to discrete spaces.

Spectral properties of the OSRC-preconditioned EFIE

Ignacia Fierro–Piccardo
Department of Mathematics, University College London, UK

In [1] we have tested the preconditioning properties of OSRC-preconditioned EFIE and showed its efficiency as a preconditioner. In this presentation, we analyse in detail how both, the Electric-to-Magnetic and Magnetic-to-Electric operator become suitable operator preconditioners for the EFIE on closed surfaces, and how these compare to one of the most used preconditioning techniques, the Calderón Preconditioner. These analyses shed light on how to build or modify possible future alternative preconditioners for more complex geometries and pose limitations and advantages when implementing the preconditioned EFIE using a Boundary Element Method scheme.

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Well-posedness of first-order formulations of wave equations and discretization by space-time finite elements

Thomas Führer

Facultad de Matemáticas, Pontificia Universidad Católica de Chile, Santiago, Chile

In this talk we show how to appropriately define a space so that the operator representing the first-order system of the acoustic wave equation is an isomorphism from this space to the Lebesgue space of square-integrable functions. This result relies on well-posedness and stability of the weak and ultra-weak formulation of the second-order wave equation. Based on this novel result we define a practicalleast-squares finite element method with piecewise polynomial and globally continuous finite element functions. The least-squares functional is equivalent to the error and is decomposed into local element contributions that can be used to steer local refinements simultaneously in space and time. We report on numerical experiments for one- and two-dimensional spatial domains.
This is a joint work with Roberto González and Michael Karkulik from Universidad Técnica Federico Santa María, Valparaíso, Chile.

Space-time FEM-BEM couplings for parabolic transmission problems

Gregor Gantner

Insitut für Numerische Simulation, Universität Bonn, Germany

In this talk, we discuss stable space-time FEM-BEM couplings to numerically solve parabolic transmission problems on the full space and a finite time interval. In particular, we demonstrate coercivity of the couplings under certain restrictions and validate our theoretical findings by numerical experiments.

Space-time adaptive boundary elements for the wave equation

Heiko Gimperlein

Engineering Mathematics, Universität Innsbruck, Austria

We give an overview of our recent work on residual a posteriori error estimates and the resulting adaptive mesh refinement procedures for boundary element methods for the wave equation. Both the weakly singular and the hypersingular integral equations are considered, with a focus on space-time Galerkin approximations. Extensions to convolution quadrature are considered.

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Wavelet compressed, modified Hilbert transform in the space-time discretization of the heat equation

Helmut Harbrecht

Departement Mathematik & Informatik, Universität Basel, Switzerland

On a finite time interval $(0, T)$, we consider the multiresolution Galerkin discretization of a modified Hilbert transform $\mathcal{H}_T$ which arises in the space-time Galerkin discretization of the linear diffusion equation. To this end, we design spline-wavelet systems in $(0, T)$, consisting of piecewise polynomials of degree ≥ 1 with sufficiently many vanishing moments, which constitute Riesz bases in the Sobolev spaces $H^s_{0,}(0, T)$. These bases provide multilevel splittings of the temporal discretization spaces into "increment" or "detail" spaces of direct sum type. Via algebraic tensor-products of these temporal multilevel discretizations with standard, hierarchic finite element spaces in the spatial domain (with standard Lagrangian FE bases), sparse space-time tensor-product spaces are obtained, which afford a substantial reduction in the number of the degrees of freedom as compared to time-marching discretizations. In addition, temporal spline-wavelet bases allow to compress certain nonlocal integrodifferential operators which appear in stable space-time variational formulations of initial-boundary value problems, such as the heat equation and the acoustic wave equation. An efficient preconditioner is proposed that affords essentially linear complexity solves of the linear system of equations which results from the full and sparse space-time Galerkin discretizations.

Direct, time-domain boundary element method in Galerkin formulation for the analysis of coupled wave-body dynamics in hydrodynamic applications

Moritz Hartmann

Institute for Maritime Energy Systems, German Aerospace Center (DLR), Germany

The assessment of motions and wave loads on ships and offshore structures is relevant in various stages of the maritime products' lifetime, e.g. in the research and development of innovative concepts, in the design process, or the optimization of offshore operations. The analysis of wave-ship interaction for new concepts in the field of emission-reduced shipping can be used to find operational limits by identifying critical motions for innovative energy converter and storage solutions on ships. With the development of assistance tools for ship and offshore operations by accompanying mid-term predictions of the future ship and structure motion based on deterministic observations of the surrounding sea state, valid operational time frames can be identified. Specifically, the development of a fast and accurate potential flow method is of interest as the target of motion prediction requires a real-time capable and precise algorithm.

We present a two-dimensional, linear prototype of a time-domain boundary element approach that evaluates the wave and ship dynamics monolithically coupled. Prior focus is set on the incorporation of an efficient and accurate solver for wave dynamics in the Galerkin-type boundary integral equation framework, the definition of the mixed boundary value problem with appearing surface discontinuities, and the assembly of boundary integral operators including hypersingular integral kernels. The validity of the approach is underlined by presenting results of analytical and hydrodynamic test cases and future development steps are discussed.

Space-time least-squares FEM for convection-diffusion problems

Christian Köthe, Olaf Steinbach
Institut für Angewandte Mathematik, TU Graz, Austria

Instationary convection-diffusion problems arise in many applications, such as, e.g., pollution simulations, heat transfer problems between thin domains, or in the modelling of flow and transport problems, to name a few. In the advection-dominated case, the solutions are characterised by boundary layers, which lead to numerical instabilities and hence unphysical solutions when discretised with standard finite element methods. Known strategies to obtain stable solutions include the Streamline-Upwind Petrov–Galerkin (SUPG) method or a residual minimisation/least-squares approach. In this talk we focus on the latter approach. We will present an abstract least-squares framework that includes a built-in error estimator that can be used in a space-time adaptive refinement scheme. Furthermore, we will show that the instationary convection-diffusion equation fits into this framework and conclude with numerical examples that confirm our theoretical findings.

Boundary element method for dilute colloidal suspensions under a shear flow

Ignacio Labarca–Figueroa
Engineering Mathematics, Universität Innsbruck, Austria

We study dilute colloidal particle suspensions under a shear flow. To solve this problem, we propose boundary-integral formulations to study advection-diffusion equations in the stationary, frequency, and time-domain. The incompressible flow corresponds to a shear flow, for which there exists a fundamental solution in the time-domain. The fundamental solution for the stationary and frequency-domain problems is approximated, and accurate discretization is achieved by a singularity subtraction technique based on the fundamental solution of the heat equation. This can also be extended for space-time boundary element computations. Numerical experiments demonstrate the effectiveness of our discretization scheme. In collaboration with Heiko Gimperlein, Thomas Franosch, and Alexander Ostermann.

A Space-time reduced basis method for the wave equation

Moritz Feuerle[1], Richard Löscher[2], Olaf Steinbach[2], Karsten Urban[1]
[1]Institut für Numerische Mathematik, Universität Ulm, Germany
[2]Institut für Angewandte Mathematik, TU Graz, Austria

For phenomena modeled by parametrized partial differential equations (PPDEs), model order reduction is crucial in many applications, due to memory or time constraints in the simulation process. Although for a large class of PPDEs the reduced basis method (RBM) is applicable and can be rigorously analyzed, there are still difficulties when considering hyperbolic problems.
In this talk we will recall an abstract framework for the analysis of the RBM, covering elliptic and parabolic PPDEs, and we will show its limited applicability to hyperbolic PPDEs. In particular, we will consider the wave equation as a model problem and outline the challenges it presents in constructing a suitable reduced order model. Building up on these tasks, we propose a new RBM for the wave equation, based on a space time finite element formulation. The theoretical findings will be complemented by numerical experiments.

Operator preconditioning in boundary element methods avoiding dual mesh

Dalibor Lukáš, Zbyšek Machaczek
Department of Applied Mathematics, VSB TU Ostrava, Czech Republic

In operator preconditioning we utilize the opposite-order mapping properties of the single-layer and hyper-singular boundary integral operators. However, in 3 spatial dimensions the lowest-order discretizations of the operators by discontinous piecewise constant and continuous piecewise linear functions, respectively, do not match in terms of degrees of freedom. Therefore, a dual mesh is often introduced to discretize the single-layer operator. Unfortunately, the assembly of the preconditioner is significantly more expensive than the operator itself. In this paper we propose and analyze a novel construction of continuous piecewise polynomial basis functions to discretize the hyper-singular operator, both in 2 and 3 dimensions. We prove that it forms an optimal preconditioner to the original single-layer operator discretized by the piecewise constants. We avoid the dual mesh. The efficiency of our approach is documented by numerical experiments performed on GPUs.

p-multilevel Monte Carlo for acoustic scattering from large deviation rough random surfaces

Michael Multerer

Istituto Eulero, USI Lugano, Switzerland

We study time harmonic acoustic scattering on large deviation rough random scatterers. Therein, the roughness of the scatterers is caused by a low Sobolev regularity in the covariance function of their deformation field. The motivation for this study arises from physical phenomena where small-scale material defects can potentially introduce non-smooth deviations from a reference domain. The primary challenge in this scenario is that the scattered wave is also random, which makes computational predictions unreliable. Therefore, it is essential to quantify these uncertainties to ensure robust and well-informed design processes. While existing methods for uncertainty quantification typically rely on domain mapping or perturbation approaches, it turns out that large and rough random deviations are not satisfactory covered. To close this gap, and although counter intuitive at first, we show that the p-multilevel Monte Carlo method can provide an efficient tool for uncertainty quantification in this setting. To this end, we discuss the stable implementation of higher-order polynomial approximation of the deformation field by means of barycentric interpolation and provide a cost-to-accuracy analysis. Our considerations are complemented by numerical experiments in three dimensions on a complex scattering geometry.

A non-symmetric space-time coupling of finite and boundary element methods for a parabolic-elliptic interface problem

Tobias Kaltenbacher, Günther Of

Institut für Angewandte Mathematik, TU Graz, Austria

We consider the interface problem of the heat equation in a bounded domain and of the Laplace equation in the exterior domain. We present a coupling of a space-time formulation of the heat equation and the weakly singular integral equation of the Laplace equation and consider a conforming space-time discretization. We discuss an analysis of the proposed space-time formulation, the discretization, an implementation of the related FEM-BEM coupling, and numerical tests.

Finite element geometric calculus for elliptic problems

Stefan Kurz[1], Michael Reichelt[2], Olaf Steinbach[2]
[1]Seminar für Angewandte Mathematik, ETH Zürich, Switzerland
[2]Institut für Angewandte Mathematik, TU Graz, Austria

Geometric algebras (or Clifford algebras) provide a uniform framework for the coordinate-free representation of geometric objects and operations, such as rotations, and are well suited for physical modelling [1]. For example, Quaternions are traditionally used to represent rotations. As it turns out, they are just a special subalgebra that is easier to understand given the big picture. These structures can be used to build a geometric calculus [1,3] that has similarities to complex calculus and differential forms. In contrast to differential forms, which have recently gained attention in the context of finite element methods [2], geometric algebras contain an inner product and thus a metric from the outset. In the context of PDEs, this can greatly simplify the notation, since a metric is still necessary at some point. In this talk, the necessary basics of geometric algebras will be presented and it will be shown how they can be used to solve elliptic PDEs in the context of finite elements.

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Anisotropic wavelet matrix compression of integral operators

Helmut Harbrecht, Remo von Rickenbach
Departement Mathematik & Informatik, Universität Basel, Switzerland

Consider an integral operator equation $\mathcal{L}u = f$ posed on the unit square $\square := [0,1]^2$ or a smooth manifold $\Gamma \subset \mathbb{R}^3$. It is assumed that $\mathcal{L} : H^q \to H^{-q}$ is continuous and uniformly elliptic, where for sufficiently smooth u there holds

$$\mathcal{L}u(\boldsymbol{x}) = \int \kappa(\boldsymbol{x}, \boldsymbol{y}) u(\boldsymbol{y}) \mathsf{d}\boldsymbol{y}.$$

In particular, we assume that the kernel κ is asymptotically smooth of order $2q$, that is,

$$\left|\partial_{\boldsymbol{x}}^{\boldsymbol{\alpha}} \partial_{\boldsymbol{y}}^{\boldsymbol{\beta}} \kappa(\boldsymbol{x}, \boldsymbol{y})\right| \lesssim \|\boldsymbol{x} - \boldsymbol{y}\|^{-(2+2q+|\boldsymbol{\alpha}|+|\boldsymbol{\beta}|)}, \quad 2 + 2q + |\boldsymbol{\alpha}| + |\boldsymbol{\beta}| > 0.$$

Given a wavelet basis $\Psi = \{\psi_\lambda : \lambda \in \nabla\}$ of H^q, we ask the following question: How many basis functions are necessary to approximate the unknown solution u up to a given precision ε? In other terms, what is the smallest N such that there exists a finite index set $\Lambda \subset \nabla$ with $|\Lambda| \leq N$ and $\|u - u_\Lambda\|_{H^q} \leq \varepsilon$?
In this talk, we will charaterise the function spaces which can be approximated with N terms at the rate N^{-s} if the underlying basis set Ψ is of *anisotropic* nature. Moreover, we will discuss under which circumstances the solution u of the operator equation $\mathcal{L}u = f$ can be approximated at the same rate as if full knowledge on the function u was provided. Therefore, we will have a brief look at the concept of $s^\star$-*compressibility* and the difficulties arising from the anisotropic structure of the wavelet functions.

3D-ACA accelerated time domain boundary element method: FMM and H-matrix based approaches

Martin Schanz
Institute of Applied Mechanics, TU Graz, Austria

The time domain Boundary Element Method (BEM) for the homogeneous wave equation with vanishing initial conditions is considered. The generalized convolution quadrature method (gCQ) developed by Lopez-Fernandez and Sauter [3] is used for the temporal discretisation. The spatial discretisation is done classically using low order shape functions. A collocation approach is applied for the Dirichlet problem and a Galerkin approach for the Neumann problem.

Essentially, the gCQ requires to establish boundary element matrices of the corresponding elliptic problem in Laplace domain at several complex frequencies. Consequently, an array of system matrices is obtained. This array of system matrices can be interpreted as a three-dimensional array of data which should be approximated by a data-sparse representation. The multivariate Adaptive Cross Approximation (3D-ACA) [1] can be applied to get a data sparse representation of these three-dimensional data arrays. Adaptively, the rank of the three-dimensional data array is increased until a prescribed accuracy is obtained. On a pure algebraic level it is decided whether a low-rank approximation of the three-dimensional data array is close enough to the original matrix. Within the data slices corresponding to the BEM calculations at each frequency either the standard H-matrices approach with ACA [2] or a fast multipole (FMM) approach can be used. The third dimension of the data array represents the complex frequencies. Hence, the algorithm makes not only a data sparse approximation in the two spatial dimensions but detects adaptively how much frequencies are necessary for which matrix block.

In the presentation, this methodology is recalled and both versions either using H-matrices in the slices or FMM will be compared. The study is numerically performed at selected examples as the mathematical analysis gives the same complexity. Nevertheless, the performance of the algorithm differs.

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A mixed approximation of the boundary element method for linear elasticity

Christina Schwarz
Chair of Scientific Computing, University of Bayreuth, Germany

Using the boundary element method to solve mixed boundary value problems generally results in non-sparse matrices. To reduce storage, we approximate the involved operators by hierarchical matrices [1].
Furthermore, to speed up simulations we would like to replace the Galerkin method by collocation. As the hypersingular integral operator is not defined in the case of collocation, we will use a mixed approximation which was introduced by O. Steinbach for the Laplace equation [2].
With the help of the Steklov-Poincaré operator a coupled saddle point problem can be derived only involving single and double layer potential operators. To ensure stability, two nested grids need to be combined for the discretization of the integral operators.
Our aim is the application of this mixed formulation to the Lamé equation from linear elasticity. In this work, we compare the results with the standard symmetric formulation and examine the influence of different parameters.

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Space-time finite element methods in thermoelasticity

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In this talk we first review space-time variational formulations for parabolic and hyperbolic initial boundary value problems. This includes formulations in Bochner and anisotropic Sobolev spaces, also using a modified Hilbert transformation, and first order systems. We then apply these results to analyze space-time variational formulations for the hyperbolic-parabolic system of thermodynamics. When eliminating the temperature, we end up with a Schur complement system where the coupling term is non-negative. First numerical results are given.

Higher order non-conforming FE/BE coupling for 2 dim. eddy current problems

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We study a non-conforming FEM/BEM coupling for a 2 dim. eddy current problem. This transmission problem is decribed by the curl curl-grad div operator in the interior bounded domain coupled to a scalar Laplace problem in the exterior domain. For the interior problem we use a stabilized, non-conforming higher order discretization in which the inter-element continuities are weakly enforced. The coupling to the exterior problem is done via the exterior Poincaré–Steklov operator. An a priori error analysis is given for h- and p-refinements and graded meshes. On triangular meshes optimal a priori error estimates are proven under sufficiently high Sobolev regularity of the solution. Numerical experiments confirm the theoretical results and demonstrate the effect of graded meshes on the order of convergence.

On the construction of the Stokes flow in a domain with cylindrical ends

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Convergence of Calderón residuals

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Convergence rates for Galerkin discretizations of boundary integral equations are usually available in fractional and sometimes even negative order spaces. Because of this, when one wishes to debug a BEM code, one chooses a reference solution and typically uses some of the Galerkin matrices to measure the error, exploiting the norm equivalence between the solution space of the boundary integral equation and the energy norm of the operator. However, this method only works if the implementation of the Galerkin matrices is done correctly, which can be hard to verify.
In this talk, we present a tool to validate the implementation of boundary integral operators that circumvents this problem. For this, we compute expected convergence rates for residuals based on the Calderń identities for general differential operators. These rates can be used to validate the implementation of boundary integral operators. Our estimates are in standard infinity and Euclidean vector norms, thus avoiding the use of hard-to-compute norms. We illustrate this with three examples: the Laplacian, time-harmonic Maxwell's equations, and the Hodge-Helmholtz equation.

Space-time BEM for the wave equation for flat objects

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In this talk, we consider a space-time boundary element method for the wave equation based on the single layer operator. We start with an overview of boundary integral equations and their discretizations for the wave equation. Next, a new approach is introduced for the case of a flat screen. We introduce new space-time Sobolev spaces by Fourier representations, and we present their most important properties. Applying the (classical) Hilbert transform leads to a coercive and continuous single layer operator in a new space-time Sobolev space. Hence, a new space-time variational formulation for the wave equation in the framework of the Lax–Milgram lemma is derived. Thus, any conforming discretization is unconditionally stable. Based on this, we present a new space-time boundary element method for the wave equation. In the last part of the talk, numerical examples are shown.

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