Technische Universität Graz

Workshop on

Domain Decomposition Solvers for Heterogeneous Field Problems

Söllerhaus, 2.–6.6.2010

A. Klawonn, U. Langer, L. F. Pavarino,
O. Steinbach, O. B. Widlund (eds.)

**Berichte aus dem
Institut für Numerische Mathematik**

Book of Abstracts 20010/8

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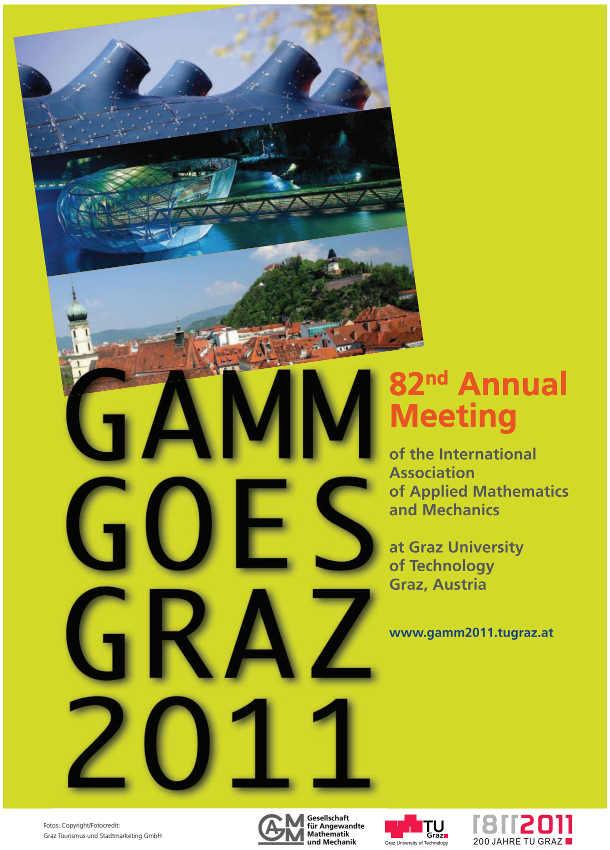
Program

Wednesday, June 2, 2010	
15.00–16.25	Coffee
16.25–16.30	Opening
16.30–17.15	M. Dryja (Warsaw) Additive average Schwarz method for discretization of elliptic problems with highly discontinuous coefficients
17.15–18.00	M. Discacciati (Lausanne) A domain decomposition framework for modeling dimensionally heterogeneous problems
18.00–18.30	M. Jarosova (Ostrava) FETI–DP averaging for the solution of linear elasticity contact problems
18.30	Dinner
Thursday, June 3, 2010	
9.00–9.45	L. F. Pavarino (Milano) BDDC preconditioners for spectral element discretizations of almost incompressible elasticity in three dimensions
9.45–10.30	H. H. Kim (Sung Kyun Kwan) A two–level nonoverlapping Schwarz algorithm for the Stokes problem without primal pressure unknowns
10.30–11.00	Coffee
11.00–11.45	O. Rheinbach (Essen) FETI–DP domain decomposition solvers in the simulation of soft biological tissues
11.45–12.30	J. Lee (Oak Ridge) Comparison of two domain decomposition methods for a linearized contact problem
12.30	Lunch
14.00–14.45	M. Kolmbauer (Linz) A frequency–robust solver for time–harmonic parabolic problems with application to the eddy current problem
14.45–15.30	M. Windisch (Graz) BETI methods for acoustic scattering in unbounded domains
15.30–16.00	Coffee
16.00–16.45	S. Weißer (Saarbrücken) Adaptive FEM with local Trefftz ansatz functions on general polygonal meshes
16.45–17.30	C. Hofreither (Linz) A non–standard finite element method based on boundary integral operators
17.30–17.45	Break
17.45–18.30	M. Sarkis (Worcester) Boundary layer technical tools for domain decomposition methods
18.30	Dinner

Friday, June 4, 2010	
9.00–9.45	R. Kornhuber (Berlin) Coupled surface and saturated/unsaterated ground water flow in heterogeneous media
9.45–10.30	D. Copeland (College Station) Domain decomposition for two-phase flow in high–contrast porous media
10.30–11.00	Coffee
11.00–11.45	J. Schöberl (Wien) A BDDC preconditioner for hybrid discontinuous Galerkin methods
11.45–12.30	M. Neumüller (Graz) A hybrid DG finite element method in space and time
12.30	Lunch
13.30–18.00	Hiking Tour
18.30	Dinner
Saturday, June 5, 2010	
9.00–9.45	J. Zou (Hong Kong) Multilevel reconstruction in inverse obstacle scattering
9.45–10.30	A. Langer (Linz) Domain decomposition methods for total variation minimization
10.30–11.00	Coffee
11.00–11.45	O. B. Widlund (New York) Domain decomposition algorithms and irregular subdomains
11.45–12.30	T. Kozubek (Ostrava) Numerically and parallely scalable FETI based algorithms for contact problems of mechanics and their powerful ingredients
12.30	Lunch
14.00–14.45	Z. Dostal (Ostrava) Theoretically supported scalable algorithms for contact problems
14.45–15.30	S. Vanis (Essen) Numerical simulation of finite micromorphic elasticity using FETI–DP domain decomposition methods
15.30–16.00	Coffee
16.00–16.45	E. P. Stephan (Hannover) Preconditioners for the hp version boundary element method
16.45–17.30	H. Yang (Linz) Numerical simulation of fluid–structure interaction problems on hybrid meshes with algebraic multigrid methods
17.30–17.45	Break
17.45–18.30	D. Feldengut (Wien) Finite element methods for domain decomposition with non–matching meshes
18.30	Dinner

Sunday, June 6, 2010	
9.00–9.45	A. Klawonn (Essen) Coarse space by projection in iterative substructuring methods
9.45–10.30	Closing and Coffee



April 18–21, 2011

Domain decomposition for two-phase flow in high–contrast porous media

D. Copeland, Y. Efendiev
Texas A&M University, College Station, USA

The modeling of multi–phase flow in porous media involves computation of mass conservative velocity fields by solving second-order elliptic PDEs. In applications to subsurface flows, the media properties are highly heterogeneous with disparate variations across spatial scales and the values of the permeability field. In this talk, we present a domain decomposition method for a mixed finite element formulation, which ensures that the convergence rate of an iterative Krylov solver is independent of contrast in the coefficient. This is done by automatically constructing a minimal coarse space by including velocity fields identified via local eigenvalue problems.

A domain decomposition framework for modeling dimensionally heterogeneous problems

P. J. Blanco[1], M. Discacciati[2], A. Quarteroni[2,3]
[1]LNCC Petropolis, Brasil
[2]EPF Lausanne, Switzerland
[3]Politecnico di Milano, Italy

In the last decade, there has been an increasing interest in the use of dimensionally heterogeneous representations of different physical systems. This so–called geometrical multiscale modeling has been applied successfully to represent physical phenomena arising in different fields such as, e.g., fluid–dynamics and structural modeling.
The appealing aspect of such an approach is that it is possible to account for the interactions between different geometrical scales in a given system. For instance, in the context of the cardio–vascular system, this allows for the integrated modeling of the blood flow, taking into account the interplay between the global systemic dynamics and the complex local blood flow behavior.
In this talk we present a general theoretical framework for coupling dimensionally heterogeneous partial differential equations and we provide some guidelines for the abstract well–posedness analysis of such problems both in the continuous and in the discrete cases. Moreover, we show how to construct suitable partitioning methodologies in the context of domain decomposition methods. In particular, we discuss some alternative possibilities to those encountered in the classical domain decomposition literature, specifically devised for the dimensionally–heterogeneous case.
Finally, we present some numerical results to illustrate the effectiveness of our approach.

Theoretically supported scalable algorithms for contact problems

Z. Dostál, T. Kozubek, M. Sadopwská, V. Vondrák,
A. Markopoulos, T. Brzobohatý
VŠB Technical University of Ostrava, Czech Republic

We first briefly review the TFETI/TBETI (total finite/boundary element tearing and interconnecting) based domain decomposition methodology adapted to the solution of 2D and 3D multibody contact problems of elasticity, including friction and classical optimal estimates. Recall that TFETI differs from the classical FETI or FETI2 by imposing the prescribed displacements by the Lagrange multipliers and treating all subdomains as "floating".
Then we present our in a sense optimal algorithms for the solution of the resulting quadratic programming problems. The unique feature of these algorithms is their capability to solve the class of convex quadratic programming problems with homogeneous equality constraints and relevant separable inequality constraints in $O(1)$ iterations provided the spectrum of the Hessian of the cost function is in a given positive interval.
Finally we put together the above results to develop scalable algorithms for the solution of both coercive and semi-coercive variational inequalities [1]. A special attention is paid to the construction of an initial approximation which is not far from the solution, so that the above results guarantee that the cost of the solution increases nearly proportionally with the dimension of the discretized problem. We illustrate the results by numerical experiments and by the solution of difficult real world problems. Our discussion will cover both frictionless problems and problems with friction. We conclude by a brief discussion of possible improvements and generalizations.

References

[1] Z. Dostál, T. Kozubek, V. Vondrák, T. Brzobohatý, A. Markopoulos, Scalable TFETI algorithm for the solution of multibody contact problems of elasticity. Int. J. Numer. Methods Eng., DOI: 10.1002/nme.2807, 2009.

Additive average Schwarz method for discretization of elliptic problems with highly discontinuous coefficients

M. Dryja
Warsaw University, Poland

In the talk the second order elliptic problems with highly discontinuous coefficients are considered. The problems are discretized by two methods: 1) a continuous finite element method (FEM) and 2) a composite discretization given by a continuous FEM inside the substructures and discontinuous Galerkin (DG) method across the boundaries of these substructures. We design and analyze parallel algorithm for the resulting discretizations. These algorithms are additive Schwarz methods (ASMs) with special coarse spaces spanned by functions that are almost piecewise constant with respect to substructures for the first discretization and piecewise constant for the second one. It is established that the condition numbers of the preconditioned systems do not depend on the jumps of the coefficients across the substructure boundaries and outside of a thin layer along the substructures boundaries. The algorithms are very well suited for parallel computations.
The results presented are a joint work with Marcus Sarkis (WPI, USA).

Finite Element Methods for Domain Decomposition with Non–Matching Meshes

D. Feldengut, K. Hollaus, J. Schöberl
TU Wien, Austria

Domain decomposition in finite element methods allows the use of independent meshes for different parts of the model. This facilitates the solution of large scale problems or problems with moving parts, and enables efficient computation through parallelization.
This talk outlines a method of domain decomposition for the Poisson problem related to Nitsche's approach to the approximation of Dirichlet boundary conditions. The method allows for non-overlappping, non-matching meshes discretizing the subdomains by introducing a hybrid variable on the interface to glue the subdomains together. It yields a positive definite global system, and stability is proven in the energy norm.
The function space for the hybrid variable can be chosen with great flexibility, which simplifies practical implementation considerably. As a basis for the interface space, B-Splines were chosen. B-Splines can be computed to any given order and smoothness with the De-Boor iteration, benefitting numerical integration. The locality of B-Splines (they are non–zero on a limited interval) minimizes coupling in the linear system.
Numerical tests were conducted to demonstrate the theoretical results.

A non–standard finite element method based on boundary integral operators

C. Hofreither

Johannes Kepler University Linz, Austria

We present a non-standard Finite Element Method that is based on the use of boundary integral operators and that permits polyhedral element shapes as well as grids with hanging nodes. The method can be interpreted as a local Trefftz method, i.e., the method employs element-wise PDE-harmonic trial functions. The construction principle requires the explicit knowledge of the fundamental solution of the partial differential operator, but only locally in every polyhedral element. This allows us to solve PDEs with elementwise constant coefficients. In this talk we consider the diffusion equation as a model problem, but the method can be generalized to convection-diffusion-reaction problems and to systems of PDEs like the linear elasticity system with element-wise constant coefficients.
We give rigorous error estimates for the three-dimensional case under quite general assumptions on the geometrical properties of the elements.

FETI–DP averaging for the solution of linear elasticity contact problems

M. Jarošová[1], Z. Dostál[1], A. Klawonn[2], O. Rheinbach[2]
[1]VŠB–Technical University of Ostrava, Czech Republic
[2]Universität Duisburg–Essen, Germany

We consider preconditioning strategies of the algorithms for the solution of linear elasticity contact problems. We are interested especially in the preconditioning strategies which result in the improved bounds on the rate of convergence. Let us point out that this goal cannot be achieved by a variant of the preconditioning in face, since such preconditioning affects only the linear steps of the algorithm, but not the nonlinear steps.
We shall focus on two strategies exploiting the edge averages for FETI–DP methods. The first one is a preconditioning by conjugate projector. In this case, the Lagrange multipliers corresponding to the variables of the coinciding edges are aggregated. The second one, an explicit transformation of basis, uses edge averages, which are introduced as new, additional primal variables.
For a special case, it is shown that both methods iterate in the same space and thus have the same rate of convergence. This theoretical result is confirmed by the solution of a model boundary variational inequality. We also give some results of the numerical experiments from 2D linear elasticity, where the improvement of the rate of convergence is illustrated.

References

[1] M. Jarošová, A. Klawonn, O. Rheinbach: Projector preconditioning and transformation of basis in FETI–DP algorithms for contact problems, submitted.

[2] M. Domorádová (Jarošová), Z. Dostál: Projector preconditioning for partially bound–constrained quadratic optimization. Numer. Linear Algebra Appl. 14 (2007) 791–806.

A two–level nonoverlapping Schwarz algorithm for the Stokes problem without primal pressure unknowns

Hyea Hyun Kim

Sung Kyun Kwan University, Korea

A FETI–DP algorithm without primal pressure unknowns has been developed by the authors for the Stokes problem. The primal counterpart of the FETI–DP algorithm is developed and analyzed. It turns out to be a two–level Schwarz algorithm with the coarse problem obtained from only the primal velocity unknowns. Numerical results are included.

Coarse space by projection in iterative substructuring methods

A. Klawonn
Universität Duisburg–Essen, Germany

The choice of the coarse space is of vital importance to the numerical scalability of domain decomposition methods for elliptic partial differential equations. An efficient implementation of the coarse problem is necessary to obtain parallel scalability on massively parallel systems.
In this talk we will discuss certain choices for coarse spaces in the dual-primal Finite Element Tearing and Interconnecting (FETI-DP) method with a special emphasis on their implementation. For three dimensional problems, edge and/or face constraints, either implemented by using optional Lagrange multipliers or combined with a transformation of basis, have been used to build the coarse problem. Recently, as an alternative or in addition, projector preconditioning has been used to enhance the FETI-DP coarse problem for contact problems; see also the talk by Marta Jarosova. In this talk we will discuss several new results for this approach. This work is motivated by an earlier joint project with Marta Jarosova and Oliver Rheinbach. The new results are obtained in joint work with Oliver Rheinbach.

A frequency–robust solver for time–harmonic parabolic problems with application to the eddy current problem*

M. Kolmbauer, U. Langer
Johannes Kepler Universität Linz, Austria

In many practical applications, for instance in computational electromagnetics, the excitation is time-harmonic. Due to the time-harmonic excitation, the resulting periodicity of the solution can be capitalized to switch from the time domain to the frequency domain. This allows us to replace the expensive time-integration procedure by the solution of a simple linear elliptic system for the amplitudes belonging to the sine- and to the cosine-excitation. The fast solution of the corresponding linear system of finite element equations is crucial for the competitiveness of this method. J. Schöberl and W. Zulehner proposed a new parameter-robust MinRes preconditioning technique for saddle point problems (*SIAM J. Matrix Anal. Appl., 29(3): 752-773, 2007*). This method allows us to construct a frequency-robust MinRes solver. The construction of this solver is outlined for a parabolic initial boundary value problem.

The generalization of this preconditioned MinRes solver to linear time-harmonic eddy current problems in electromagnetics is not straight forward. Due to the non-trivial kernel of the **curl** operator, we have to perform an exact regularization of the frequency domain equations in order to provide a theoretical basis for the application of the MinRes preconditioner.

We also consider the case of domains consisting of parts with positive conductivity and parts with conductivity equals zero. In this setting we can take advantage of the fact, that the exact regularization in the conducting regions also acts as a gauging condition in the non-conducting regions. Hence we end up with a well posed formulation in H(**curl**).

Finally we discuss the application of this solver to linear parabolic initial boundary value problems with non-harmonic excitation and non-linear problems.

*The authors acknowledge the support by the Austrian Science Fund (FWF) under the grant P19255.

Coupled surface and saturated/unsaturated ground water flow in heterogeneous media

R. Kornhuber
Freie Universität Berlin, Germany

Richards equations for saturated/unsaturated groundwater flow is based on state equations relating saturation to capillary pressure. The numerical solution of the resulting degenerate parabolic problems typically suffers from strong nonlinearities and ill-conditioning in the presence of strongly varying saturation. As a remedy, we suggest a solver–friendly discretization based on Kirchhoff transformation which can be reinterpreted in physical variables in terms of suitable quadrature rules. In this way ill-conditioning is separated from the numerical solution process. We show convergence and provide error estimates. This approach is extended to heterogeneous state equations by domain decomposition methods based on nonlinear transmission conditions. We suggest suitable coupling conditions of ground water flow and shallow water equations for surface water flow. The coupled system is solved by a Dirichlet–Neumann–type iteration accounting for the multiple time scales. The performance of the algorithms is illustrated by numerical computations.

Numerically and parallely scalable FETI based algorithms for contact problems of mechanics and their powerful ingredients*

T. Kozubek, Z. Dostal

VSB TU Ostrava, Czech Republic

We first briefly review the TFETI based domain decomposition methodology adapted to the solution of 2D and 3D multibody contact problems of elasticity. Recall that TFETI imposes the prescribed displacements by the Lagrange multipliers, so that all the subdomains are floating and their kernels are a priori known. Then we show that the natural coarse grid of the rigid body motions introduced by Farhat, Mandel, and Roux defines a projector to the subspace of Lagrange multipliers with the solution. Moreover, the preconditioning by the projector reduces the condition number of the dual Schur complement so that it is independent on the discretization parameter h and accelerates also the non–linear steps.

Then we present our in a sense optimal algorithms [1] for the solution of resulting quadratic programming problems together with their powerful ingredients such as stable pseudoinverse computation, fixing rigid body motions, domain decomposition postprocessing, parallel implementation, etc. The unique feature of these algorithms is their capability to solve the class of quadratic programming problems with spectrum in a given positive interval in O(1) iterations. The theory yields the error bounds that are independent on conditioning of constraints and the results are valid even for linearly dependent equality constraints.

Finally, we put together the above results to develop scalable algorithms for the solution of both coercive and semi-coercive variational inequalities (see [2] and [3]). Rather surprisingly, the results are qualitatively the same as the classical results on scalability of FETI for linear elliptic problems. We give results of numerical experiments with parallel solution of both coercive and semicoercive 2D and 3D contact problems discretized by up to more than 10 million of nodal variables to demonstrate that the scalability can be observed in computational practice. The power of the results is demonstrated also by the solution of difficult real world problems as analysis of the roller bearing of wind generator, analysis of the yielding clamp connection of steel arched supports, etc.

References

[1] Z. Dostal: Optimal Quadratic Programming Algorithms, with Applications to Variational Inequalities. Springer, New York, 2009.

[2] Z. Dostal, T. Kozubek, V. Vondrak, T. Brzobohaty, A. Markopoulos: Scalable TFETI algorithm for the solution of multibody contact problems of elasticity. Internat. J. Numer. Methods Engrg., DOI: 10.1002/nme.2807, 2009.

[3] Z. Dostal, T. Kozubek, P. Horyl, T. Brzobohaty, A. Markopoulos: Scalable TFETI algorithm for two dimensional multibody contact problems with friction. J. Comput. Appl. Math., submitted, 2009.

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Domain decomposition methods for total variation minimization

A. Langer
RICAM, Linz, Austria

Several methods have been recently presented in the literature for performing very efficiently total variation minimization, e.g., for image processing problems of small or medium size. However, because of their iterative–sequential formulation, none of the mentioned methods is able to address in real–time, or at least in an acceptable computational time, extremely large problems, such as 4D imaging (spatial plus temporal dimensions) from functional magnetic–resonance in nuclear medical imaging, astronomical imaging or global terrestrial seismic tomography. For such large scale simulations we need to address methods which allow us to reduce the problem to a finite sequence of sub–problems of a more manageable size, perhaps computable by one of the methods above. With this aim we present in this talk the complicated problem of formulating efficient overlapping and nonoverlapping domain decomposition methods for total variation minimization. These are the first successful attempts of addressing such a strategy for the nonlinear, nonadditive, and nonsmooth problem of total variation minimization. In particular we are able to present a sketch of their convergence proof. We show also several convincing numerical experiments, with successful applications of the algorithms for the restoration 1D and 2D signals in interpolation/inpainting problems respectively, and in a compressed sensing problem, for recovering piecewise constant medical–type images from partial Fourier ensembles.

Comparison of two domain decomposition methods for a linearized contact problem

Jungho Lee
Oak Ridge National Laboratory, USA

A three–level domain decomposition is considered. Bodies in contact with each other are divided into subdomains, which in turn are the union of elements. Using an approach based purely on FETI (finite element tearing and interconnecting) algorithms with only Lagrange multipliers as unknowns, which has been developed by the engineering community, does not lead to a scalable algorithm with respect to the number of subdomains in each body. We present a proof that such a method has a condition number which depends linearly on the number of subdomains across each body and logarithmically on the number of elements across each subdomain. We also propose a new method based on the saddle point formulation of the FETI methods with both displacement vectors and Lagrange multipliers as unknowns. The resulting system is solved with a block–diagonal preconditioner which combines the one–level FETI and the BDDC (balancing domain decomposition by constraints) methods. We show that this new method is scalable with respect to the number of subdomains.

A hybrid DG finite element method in space and time*

M. Neumüller, O. Steinbach
TU Graz, Austria

As a model problem we consider the time dependent heat equation. For a spatial domain $\Omega \in \mathbb{R}^d, d = 1, 2, 3$ the heat equation will be discretized in the space time cylinder $Q = \Omega \times (0, T)$. This approach results in a large system of linear equations. To handle such a approach allows the application of parallel solution algorithms to solve the large system of linear equations.

*The authors acknowledge the support by the Austrian Science Fund (FWF) under the grant SFB F32.

BDDC preconditioners for spectral element discretizations of almost incompressible elasticity in three dimensions

L. F. Pavarino[1], O. B. Widlund[2], S. Zampini[1]
[1]Universita' di Milano, Italy
[2]Courant Institute of Mathematical Sciences, New York, USA

BDDC algorithms are constructed and analyzed for the system of almost incompressible elasticity discretized with Gauss-Lobatto-Legendre spectral elements in three dimensions. Initially mixed spectral elements are employed to discretize the almost incompressible elasticity system, but a positive definite reformulation is obtained by eliminating all pressure degrees of freedom interior to each subdomain into which the spectral elements have been grouped. Appropriate sets of primal constraints can be associated with the subdomain vertices, edges, and faces so that the resulting BDDC methods have a fast convergence rate independent of the almost incompressibility of the material. These primal constraints can be point constraints and averages and moments over edges and/or faces of the subdomains. A convergence rate bound can be proved if the set of primal constraints works well in the compressible case and, in addition, a no net flux condition is satisfied across the boundary of each subdomain. In particular, the condition number of the BDDC preconditioned operator is shown to depend only weakly on the polynomial degree n, the ratio H/h of subdomain and element diameters, and the inverse of the inf-sup constants of the subdomains and the underlying mixed formulation, while being independent of the number of subdomains (scalability) and of the Poisson ratio and Young's modulus of the material considered (robustness). These results also apply to the related FETI-DP algorithms defined by the same set of primal constraints. Numerical experiments carried out on parallel computing systems confirm these results and illustrate the effects of the choice of the primal constraints.

FETI–DP domain decomposition solvers in the simulation of soft biological tissue

O. Rheinbach
Universität Duisburg–Essen, Germany

The mechanical behavior of soft biological tissue can be described by anisotropic, hyperelastic elasticity models. Such models typically incorporate a term to enforce an almost incompressibility condition; within an iterative solver environments, this can have a severe impact on the convergence. Different choices to cope with this challenge within a Newton–Krylov–FETI approach are discussed. This is a joint project with Axel Klawonn, Sarah Brinkhues, Dominik Brands, Jörg Schröder.

Boundary Layer Technical Tools for Domain Decomposition Methods

M. Sarkis
Worcester Polytechnique Institute, USA

We consider traces and discrete harmonic extensions on thin boundary layers. We discuss estimates on how to control the $H^{1/2}-$ or $H_{00}^{1/2}-$ boundary norm of a finite element function by its energy in a thin layer and vice versa, how to control the energy of discrete harmonic functions in a layer by the $H^{1/2}$ or $H_{00}^{1/2}$ norm on the boundary. We present applications and numerical results of these results applied to FETI-DP methods in the presence of high-contrast media inclusions and/or inexact solvers. This is a joint work with Prof. Maksymilian Dryja.

A BDDC Preconditioner for Hybrid Discontinuous Galerkin Methods

J. Schöberl
TU Wien, Austria

We first explain hybrid DG methods for the Poisson equation as a simple model problem for more advanced applications. Then we show how to apply the BDDC domain decomposition preconditioner in the DG and HDG framework. Computational examples demonstrate the performance of this approach.

Preconditioners for the hp–version boundary element method

E. P. Stephan
Leibniz Universität Hannover, Germany

We present new results from [1, 2] on various Schwarz methods for the h and p versions of the boundary element method applied to prototype first kind integral equations on surfaces. When those integral equations (weakly/hypersingular) are solved numerically by the Galerkin boundary element method, the resulting matrices become ill-conditioned. Hence, for an efficient solution procedure appropriate preconditioners are necessary to reduce the numbers of CG-iterations. In the p version where accuracy of the Galerkin solution is achieved by increasing the polynomial degree the use of suitable Schwarz preconditioners (presented in the paper) leads to only polylogarithmically growing condition numbers. For the h version where accuracy is achieved by reducing the mesh size we present a multi–level additive Schwarz method which is competitive with the multigrid method.

References

[1] N. Heuer, F. Leydecker, E. P. Stephan: Iterative substructering method for the p–version of the BEM on triangular meshes. Num. Meth. Partial Diff. Eq. 23 (2007) 879–903.

[2] M. Maischak, F. Leydecker, E. P. Stephan: Some Schwarz methods for integral equations on surfaces – h and p versions, Comput. Vis. Sci. 8 (2005) 211–216.

Numerical simulation of finite micromorphic elasticity using FETI–DP domain decomposition methods

A. Klawonn, P. Neff, O. Rheinbach, S. Vanis
Universität Duisburg–Essen, Germany

A minimization problem which models geometrically exact generalized continua of micromorphic type is considered. The arising problem is a two field problem in the elastic deformation φ of a given body Ω and an additional tensorial field P. Here, the tensorial field can be used to model further features needed for a more reliable descriptions of solids.

To solve this problem a staggered algorithm is introduced. This algorithm decouples the original problem into two separate problems. In each of the subproblems one variable, i.e., φ or P, is kept fixed while the problem is solved for the remaining variable, i.e., P or φ, respectively; the problem when the tensorial field P is kept fixed, is denoted as P–elasticity. Both problems are discretized with finite elements. Numerical results for the staggered approach are presented for a cubic and a cylindrical geometry.

Furthermore the subproblem of P–elasticity is considered in more detail. Here, we obtain a model which reminds of linear elasticity and in fact reduces to standard linear elasticity if the tensorial field is chosen to be the identity, i.e., $P = \mathrm{Id}$. Here, the strain tensor $\epsilon_P := \mathrm{sym}(P^{-1}\nabla u)$ is redefined to include the tensorial field P. For this problem the FETI–DP method is formulated and a convergence estimate is provided for the special case that $P^{-\top} = \nabla\psi$ is a gradient. The function ψ is considered to be at most piecewise quadratic such that we can represent it with our nodal basis functions exactly. Also a quadratic-logarithmical dependence of the condition number on the number of unknowns of each subdomain is shown. The dependence of the constants of the bound on P is highlighted. Numerical experiments confirming the theoretical findings are presented. Further results for examples which are not covered by the theoretical estimates and which are also promising are presented as well.

Adaptive FEM with local Trefftz ansatz functions on general polygonal meshes

S. Weißer
Universität des Saarlandes, Germany

We discuss a special finite element method that solves the stationary isotropic heat equation with Dirichlet boundary conditions on arbitrary polygonal and polyhedral meshes. The method uses a space of locally harmonic ansatz functions to approximate the solution of the boundary value problem. Due to this choice, we obtain a variational formulation on the skeleton of the domain. This formulation contains one Steklov-Poincaré-Operator for each element. These operators are constructed by means of boundary integral formulation. Therefore, the proposed finite element method can be used on general polygonal non-conform meshes. Hanging nodes are treated quite naturally and the material properties are assumed to be constant on each element. In a second step we focus on uniform and adaptive mesh refinement. One important point is the treatment of these arbitrary elements. We propose a method to refine polygonal bounded elements which are convex. In order to do adaptive mesh refinement it is essential to look at a posteriori error estimates. Standard methods are based on triangular or quadrilateral meshes. The challenging part is to handle the arbitrary polygonal and polyhedral meshes. Therefore, we make use of functional analytic estimates to overcome these problems.

References

[1] D. Copeland, U. Langer, D. Pusch: From the Boundary Element Domain Decomposition Methods to local Trefftz Finite Element Methods on Polyhedral Meshes. In: Domain Decomposition Methods in Science and Engineering XVIII (M. Bercovier, M. J. Gander, R. Kornhuber, O. Widlund eds.), Springer Lecture Notes in Computational Science and Engineering, vol. 70, pp. 315–322, 2009.

[2] S. Repin: A Posteriori Estimates for Partial Differential Equations. Walter de Gruyter, 2008.

Domain Decomposition Algorithms and Irregular Subdomains

O. B. Widlund

Courant Institute of Mathematical Sciences, New York, USA

A number of results have been obtained, several years ago, for domain decomposition algorithms for H^1 problems in the plane, in a collaborative effort with Clark Dohrmann, Axel Klawonn, and Oliver Rheinbach. The subdomains do not have to be Lipschitz but were only required to be John or uniform domains.

In this talk, we will first consider an overlapping Schwarz subdomain with an alternative coarse space which in fact is more similar to that of the classical algorithm as defined on triangular subdomains. We will also discuss new results for H(curl) in two dimensions and raise certain issues on problems in three dimensions.

The new results have all been obtained in collaboration with Clark Dohrmann.

BETI methods for acoustic scattering in unbounded domains*

O. Steinbach, M. Windisch
TU Graz, Austria

In this talk we present a domain decomposition method for the Helmholtz equation. We use a Tearing and Interconnecting approach for the decomposition and we use boundary integral equations to solve the local subproblems. Unfortunately, the local Neumann problems which have to be solved are not for all frequencies uniquely solvable. By exchanging the Neumann interface conditions by Robin transmission conditions we can get rid of this bottle neck. If we have an unbounded domain an additional problem appears since our formulation by boundary integral operators, which leads to uniquely solvable systems in the interior, lacks this property in the exterior. So we introduce a regularized formulation, closely related to CFIE's which fits to our domain decomposition approach.

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Numerical Simulation of Fluid–Structure Interaction Problems on Hybrid Meshes with Algebraic Multigrid Methods

Huidong Yang
RICAM, Linz, Austria

Fluid–structure interaction problems arise in many application fields such as flows around elastic structures or blood flow problems in arteries. One method for solving such a problem is based on a reduction to an equation at the interface, involving the so-called Steklov-Poincaré operators.
This interface equation is solved by a Newton iteration for which directional derivatives with respect to the interface perturbation have to be evaluated appropriately. One step of the Newton iteration requires the solution of several decoupled linear sub-problems in the structure and the fluid domains.
These sub–problems are spatially discretized by a finite element method on hybrid meshes containing different types of elements. For the time discretization implicit first order methods are used. The discretized equations are solved by algebraic multigrid methods for which a stabilized coarsening hierarchy is constructed in a proper way.

Multilevel Numerical Reconstruction in Inverse Obstacle Scattering*

Jun Zou
The Chinese University of Hong Kong

In this talk we shall discuss two newly proposed numerical technqiues to enhance the robustness and effectiveness of linear sampling methods for the reconstruction of scatterers in inverse obstacle scattering problems. The first technique uses the multilevel idea while the second one introduces an artificial reference object in the concerned scattering system. These technqiues will be demonstrated to be able to effectively overcome the three barriers of linear sampling methods.
This is a joint work with J. Li (ETH) and H.Y. Liu (U of Washington).

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