Technische Universität Graz

10. Workshop on

Fast Boundary Element Methods in Industrial Applications

Söllerhaus, 27.–30.9.2012

U. Langer, O. Steinbach, W. L. Wendland (eds.)

Berichte aus dem Institut für Numerische Mathematik

Book of Abstracts 2012/9

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Program

Thursday, September 27, 2012	
15.00–16.20	Coffee
16.20–16.30	Opening
16.30–17.00	S. Kurz (Tampere) Green's double forms for the Hodge–Laplacian on Riemannian manifolds with constant curvature: A case study in two dimensions
17.00–17.30	M. Schweiger (London) A new framework for boundary element forward modelling in diffuse optical tomography
17.30–18.00	H. Harbrecht (Basel) Wavelet boundary element methods for the polarizable continuum model in quantum chemistry
18.30	Dinner
Friday, September 28, 2012	
9.00–9.30	Z. Peng (Columbus) A surface integral equation domain decomposition method for solving time–harmonic Maxwell equations in $\mathbb{R}^3$
9.30–10.00	M. Peters (Basel) Comparison of fast boundary element methods on parametric surfaces
10.00–10.30	D. Amann (Graz) Electrostatic field problems with floating potentials
10.30–11.00	Coffee
11.00–11.30	E. Spindler (Zürich) Well–conditiond second kind single trace BEM for acoustic scattering at composite objects
11.30–12.00	J. Zapletal (Graz/Ostrava) Semi–analytic integration for the hypersingular operator related to the Helmholtz equation in 3D
12.00–12.30	A. Kimeswenger (Graz) Boundary control problems in unbounded domains
12.30	Lunch
15.00–15.30	Coffee
15.30–16.00	W. Smigaj (London) BEM++ – a new boundary element library
16.00–16.30	L. Kielhorn (Zürich) On single- and multi-trace implementations for scattering problems with BETL
16.30–17.00	M. Bantle (Ulm) eps–BEM: efficient p–stable boundary element methods: a MATLAB software package
17.00–17.30	Break
17.30–18.00	M. Faustmann (Wien) Existence of $\mathcal{H}$ matrix approximants to inverse BEM matrices
18.00–18.30	B. Kager (Graz) $\mathcal{H}$ matrix approximation to indirect time–domain BEM in elastodynamics
18.30	Dinner

Saturday, September 29, 2012	
9.00–9.30	A. Chernov (Bonn) Sparse space–time versus high order Galerkin BEM for the nonstationary heat equation
9.30–10.00	G. Of (Graz) Non–symmetric coupling of finite and boundary element methods for the heat equation
10.00–10.30	T. Führer (Wien) FEM–BEM coupling without stabilization
10.30–11.00	Coffee
11.00–11.30	C. Hofreither (Linz) FETI solvers for a BEM based finite element method
11.30–12.00	M. Karkulik (Wien) Novel inverse estimates for non–local operators
12.00–12.30	G. Unger (Graz) Coupled FE/BE eigenvalue problems for fluid–structure interaction
12.30	Lunch
13.30–18.00	Hiking Tour
18.30	Dinner
Sunday, Septmber 30, 2012	
9.00–9.30	D. Lukas (Ostrava) Parallel fast BEM for distributed memory systems
9.30–10.00	T. Traub (Graz) Solving the Lame–Navier equations using the convolution quadrature method and the directional fast multipole method
10.00–10.30	M. Feischl (Wien) Quasi–optimal adaptive BEM
10.30–11.00	Coffee
10.30–11.00	E. P. Stephan (Hannover) Boundary elements and a smoothed Nash–Hörmander iteration for the Molodensky problem
11.30–12.00	O. Steinbach (Graz) Boundary integral equations for Helmholtz boundary value and transmission problems
12.00	Closing

11. Söllerhaus Workshop on

Fast Boundary Element Methods in Industrial Applications

September 26–29, 2013

Electrostatic field problems with floating potentials

D. Amann[1], A. Blaszczyk[2], G. Of[1], O. Steinbach[1]
[1]TU Graz, Austria, [2]ABB Baden, Switzerland

For the solution of electrostatic field problems we discuss several boundary integral formulations. In particular, we present and compare two approaches on how to handle floating potentials, i.e., domains on which the potential has a constant but unknown value.

One approach is to treat the floating potential as a dielectricum with a very high relative permittivity. By doing so, one achieves an approximation of the potential which is almost constant. The second approach includes the unknown constant value of the potential on the floating domain as an Lagrangian multiplier into the formulation. As constraint the flux over the boundary of the floating potential has to vanish. The second approach results in a saddle point problem.

For both approaches an indirect formulation using the single layer potential and a Steklov–Poincaré interface formulation is considered. The strengths and weaknesses of the presented formulations shall be demonstrated with several numerical examples. Besides academical examples like spheres also real life problems of transformers are presented.

eps-BEM: efficient p-stable boundary element methods: a MATLAB software package

M. Bantle, S. Funken
Universität Ulm, Germany

We present a MATLAB software package for *hp*-Boundary Element Methods for Laplace and Lamé equations in two dimensions which allows for calculations with large polynomial degrees and features various *hp*-mesh refinement strategies.
Using Legendre polynomials and Lobatto shape functions as ansatz functions results in simple recurrence relations and gives a relation to Legendre functions of the second kind. This enables us to calculate potentials and Galerkin entries even for high polynomial degrees in an efficient way up to machine precision. Whereas in most previous publications, numerical results are only presented for polynomial degrees up to $p = 20$ our approach allows for the calculations with polynomial degrees $p > 1000$. This is achieved by a combination of forward an backward recurrence relations for the potentials and the use of high precision libraries such as mpfr and mpc for the Galerkin entries, respectively.
The core of the software package is made up of a C-library that contains procedures for the evaluation of Legendre functions and their integrals. The top-layer of the software package, which consists of the user interface and other functions the user would be interested in, is programmed in MATLAB. This combination of C and MATLAB results in a software package that provides not only very efficient and stable procedures but is also easy to use. Therefore it is useful for both teaching and research.
Some key features of eps-BEM are

- fast and stable evaluation of the potentials for large polynomial degrees,
- computation of the Galerkin entries,
- different types of error estimators,
- functions for displaying solutions,
- examples for several *hp*-adaptive mesh refinement strategies.

Sparse space-time versus high order Galerkin BEM for the nonstationary heat equation

Alexey Chernov[1], Christoph Schwab[2]

[1]Hausdorff Center for Mathematics and Institute for Numerical Simulation, University of Bonn, Germany

[2]Seminar for Applied Mathematics, ETH Zurich, Switzerland

We construct and analyze sparse tensorized space-time Galerkin discretizations for boundary integral equations resulting from the boundary reduction of nonstationary diffusion equations with either Dirichlet or Neumann boundary conditions. The approach is based on biorthogonal multilevel subspace decompositions and a weighted sparse tensor product construction. We compare the convergence behavior of the proposed method to the standard full tensor product discretizations. In particular, we show for the problem of nonstationary heat conduction in a bounded two- or three-dimensional spatial domain that low order sparse space-time Galerkin schemes are competitive with high order full tensor product discretizations (see e.g. [2]) in terms of the asymptotic convergence rate of the Galerkin error in the energy norms, under lower regularity requirements on the solution.

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Existence of $\mathcal{H}$-matrix approximants to inverse BEM matrices

M. Faustmann, J. M. Melenk, D. Praetorius
TU Wien, Austria

The matrices arising in BEM, for example, the matrix $\mathbf{V}$ for the classical simple-layer operator V, are fully populated. So are typically the relevant inverses, e.g., $\mathbf{V}^{-1}$. Various compression techniques such as $\mathcal{H}$-matrices have been developed in the past to store the BEM matrices and realize the matrix-vector-multiplication with log-linear (or even linear) complexity. The $\mathcal{H}$-matrices come with an arithmetic that includes the (approximate) inversion, (approximate) LU-decomposition etc. Numerically it has been observed that these algorithms indeed yield good approximations to the inverses of BEM matrices such as $\mathbf{V}^{-1}$, [1, 2, 3, 4].
As a first step towards mathematically understanding the success of these observations, we establish in this talk that the $\mathcal{H}$-matrix format is rich enough to permit good approximations of these inverses. As an example, we consider the simple-layer matrix $\mathbf{V}$ on quasi-uniform meshes on polygonal/polyhedral boundaries. The cluster tree is generated using the standard admissibility condition. For the resulting $\mathcal{H}$-matrix format, we show that $\mathbf{V}^{-1}$ can be approximated at an exponential rate in the block rank. The result extends corresponding results of Bebendorf & Hackbusch [1] and Börm [3] in several ways: It applies to BEM-operators instead of FEM-operators, and it avoids the needs to have fairly explicit information about the continuous inverse V^{-1}. The latter observation, in particular, results in our being able to show exponential convergence in the block rank.
Finally, an approximation in the framework of $\mathcal{H}^2$-matrices can be achieved as well.

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Quasi-optimal adaptive BEM

M. Feischl, T. Führer, M. Karkulik, D. Praetorius
TU Wien, Austria

Based on the recent work [4], we consider the weakly singular integral equation

$$V\phi = f := (1/2 + K)g \tag{1}$$

and an adaptive mesh-refining algorithm driven by some weighted residual error estimator ρ_ℓ from [1] plus data oscillation terms osc_ℓ. Here, Γ is the boundary of a polygonal Lipschitz domain $\Omega \subset \mathbb{R}^d$, $d = 2; 3$, and V and K are the simple-layer resp. double-layer potential of the 2D or 3D Laplacian.
To deal with discrete integral operators only and to thus ease the use of fast methods, we follow [2] and additionally discretize the given Dirichlet data $g \approx G_\ell$. For a boundary mesh $\mathcal{T}_\ell$, we denote by $\Phi_\ell \in \mathcal{P}^0(\mathcal{T}_\ell)$ the $\mathcal{T}_\ell$-piecewise constant Galerkin approximation

$$\langle V\Phi_\ell.\Psi_\ell\rangle_\Gamma = \langle(1/2 + K)G_\ell, \Psi_\ell\rangle_\Gamma \quad \text{for all } \Psi_\ell \in \mathcal{P}^0(\mathcal{T}_\ell),$$

where $G_\ell := P_\ell g$ is a piecewise linear, globally continuous approximation of g generated by an arbitrary, but $H^{1/2}$-stable projection P_ℓ onto $\mathcal{S}^1(\mathcal{T}_\ell)$.
In this frame, we transfer the results available for adaptive finite elements methods, see e.g. [3], to adaptive boundary element method so that the usual adaptive algorithm now has the same theoretical foundation as for adaptive finite element methods. This includes guaranteed convergence $\Phi_\ell \to \phi$ as well as quasi-optimal convergence rates for $\rho_\ell + \mathrm{osc}_\ell$. This particularly means that each convergence rate N^{-s} which is theoretically possible for some optimally chosen sequence of meshes, is in fact also guaranteed for the sequence of adaptively generated meshes.

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FEM-BEM coupling without stabilization

M. Aurada, M. Feischl, T. Führer, M. Karkulik, J. M. Melenk, D. Praetorius
TU Wien, Austria

We consider a nonlinear interface problem which can equivalently be stated via various FEM-BEM coupling methods. We consider Costabel's symmetric coupling as well as the non-symmetric Johnson-Nédélec coupling and the Bielak-MacCamy coupling. Due to constant functions in the kernel of these equations, these formulations are not elliptic and unique solvability cannot be shown directly. Available results include:

- For the symmetric coupling and certain nonlinear problems, Carstensen & Stephan (1995) proved unique solvability for sufficiently fine meshes.
- For the Johnson-Nédélec coupling and the linear Laplace and Yukawa transmission problem, Sayas (2009) proved unique solvability.
- For the Johnson-Nédélec coupling and a general class of linear problems, Steinbach (2011) introduced a stabilization to prove ellipticity of the stabilized coupling equations.

The approach of Steinbach requires pre- and postprocessing steps which involve the numerical solution of an additional integral equation with the simple-layer potential. In our talk, we present a framework based on *implicit theoretic stabilization* to prove well-posedness of nonlinear FEM-BEM coupling formulations. We build on the works of Sayas and Steinbach and introduce stabilized coupling equations which are uniquely solvable and have the same solution as the (original) continuous resp. discrete coupling equations.
With this *theoretic auxiliary problem*, we obtain unique solvability of the original coupling equations. In particular, we avoid the solution of any additional equation and corresponding pre- and postprocessing steps as well as any assumption on the mesh-size.

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Wavelet boundary element methods for the polarizable continuum model in quantum chemistry

H. Harbrecht[1], L. Frediani[2]

[1]Universität Basel, Switzerland, [2]University Tromsø, Norway

The present talk is concerned with the rapid solution of boundary integral equations which arise from solvation continuum models. We apply a fully discrete wavelet Galerkin scheme for the computation of the apparent surface charge on solvent accessible and solvent excluded surfaces. This scheme requires parametric surfaces. We therefore developed a mesh generator which automatically constructs a parametrization of the molecular surface by four-sided patches. Numerical results are presented which demonstrate the feasibility and scope of our approach.

FETI solvers for a BEM-based finite element method

C. Hofreither, U. Langer, C. Pechstein
Johannes Kepler Universität Linz, Austria

We present efficient Domain Decomposition solvers for a class of non-standard Finite Element Methods. These methods utilize PDE-harmonic trial functions in every element of a polyhedral mesh, and use boundary element techniques locally in order to assemble the finite element stiffness matrices. For these reasons, the terms BEM-based FEM or Trefftz-FEM are sometimes used. In the present talk, we show that Finite Element Tearing and Interconnecting (FETI) methods can be used to solve the resulting linear systems in a quasi-optimal and parallel manner. An important theoretical tool are spectral equivalences between FEM- and BEM-approximated Steklov-Poincare operators.

$\mathcal{H}$-matrix approximation to indirect time-domain BEM in elastodynamics

B. Kager
TU Graz, Austria

When dealing with an indirect approach for elastodynamics in time domain, e.g. with the single layer potential, the arising lower triangle Toeplitz system is based upon $\frac{1}{2}M^2$ system matrices, where M represents the overall number of timesteps. Due to causality, these matrices are sparsely populated. We present a scheme that uses $\mathcal{H}$-Matrix structure to perfom a sparse as well as a low rank approximation in between the travelling wavefronts. Thus, a further reduction of the memory consumption can be established.
We present the used methods as well as a study on memory reduction and computational time.

Novel inverse estimates for non-local operators

M. Feischl, T. Führer, M. Karkulik, J. M. Melenk, D. Praetorius
TU Wien, Austria

Inverse estimates are a means to bound stronger norms by weaker ones. Typically, they only hold on finite-dimensional spaces, and the constants involved depend on the dimension. For example, a typical inverse estimate from FEM reads

$$C\|h_{\mathcal{T}}\nabla u\|_{L_2} \leq \|u\|_{L_2} \quad \text{for all } u \in \mathcal{S}^1(\mathcal{T}), \tag{1}$$

where $\mathcal{T}$ can even be a locally refined mesh with mesh size $h_{\mathcal{T}}$. Estimates of the type (1) with C independent of $\mathcal{T}$ can be shown when the following assumptions are met: (a) it is possible to reduce the considerations to local configurations, i.e. elements or patches; (b) it is possible to use norm equivalence on finite dimensional spaces on these local configurations.
If V denotes the simple layer potential of the Laplacian, and $\mathcal{P}^0(\mathcal{T})$ is the space of piecewise constants, the interpolation theorem and the mapping properties of V yield the inverse estimate

$$\|h_{\mathcal{T}}^{1/2}\nabla V\Psi\|_{L_2} \leq C\|\Psi\|_{H^{-1/2}} \quad \text{for all } \Psi \in \mathcal{P}^0(\mathcal{T}), \tag{2}$$

on quasi-uniform meshes $\mathcal{T}$. For locally quasi-uniform meshes $\mathcal{T}$, the proof of (2) is far from obvious, as (a) and (b) cannot be used due to the non-locality of V and $\|\cdot\|_{H^{-1/2}}$. In this talk, we present an approach that is based on regularity theory for PDEs. We even allow for other boundary integral operators, high-order spaces and estimates which are explicit in the local polynomial degree. Inverse estimates like (2) are needed to prove convergence of adaptive BEM or FEM-BEM coupling [1, 3], or to prove efficiency of residual-based error estimators in BEM [2]. This talk is based on the recent work [4].

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On single- and multi-trace implementations for scattering problems with BETL

L. Kielhorn
ETH Zürich, Switzerland

Since the *Boundary Element Template Library* (BETL) has been presented throughout the former Söllerhaus Workshop we will focus on a more specific topic this time.

The talk will be concerned with some programming aspects of simple implementations for acoustic and/or electromagnetic scattering problems. After some theoretical remarks on the mathematical description of single- and multi-trace formulations via Calderón operators we will give some details on how the involved boundary integral operators are abstracted within BETL and how they are embedded into a boundary element formulation.

Finally, some numerical results as well as some ideas on preconditioning will be presented.

Boundary control problems for unbounded domains

A. Kimeswenger, O. Steinbach
TU Graz, Austria

The aim of this work is to handle boundary control problems where the considered domain is assumed to be unbounded. As the observation takes place on a bounded subset of the domain, a BEM-FEM-approach will be used to handle the resulting optimality system. The optimality system contains a primal problem, a dual problem and a first order optimality condition. It is known that a symmetric approach leads to better properties when doing numerics. Hence the optimality system will be modified to obtain a symmetric system. A standard BEM-FEM discretization will be used and some $2d$ simulations will be discussed. Sometimes it makes sense to impose box constraints for the control variable. Therefore a semi-smooth Newton method will be used to handle the constrained problem.

Green's double forms for the Hodgle–Laplacian on Riemannian manifolds with constant curvature: A case–study in two–dimensions

S. Kurz
Tampere University of Technology, Finland

In [1], boundary integral equations for Maxwell-type problems have been discussed in terms of differential forms. Such problems are governed by the equation

$$(\delta d - k^2)\omega = 0, \quad k \in \mathbb{C},$$

where d denotes the exterior derivative, δ the co-derivative, and ω is a differential form of degree p. This problem class generalizes **curl curl**- and div **grad**-types of problems in three dimensions. In the differential forms framework, kernels of integral transformations are described by double forms. The relevant Green's double form is given as fundamental solution of the Helmholtz equation,

$$(\Delta - k^2)G = \delta(X, X')I.$$

Herein, Δ is the Hodge-Laplacian, G the Green's double form, δ the Dirac delta distribution, and I the identity double form.
In contrast to the classical vector analysis formulation in Euclidean space, the differential forms-based formulation and analysis remains valid in curved spaces as well. However, this fact has not been exploited in [1].
In flat space, the Green's double form of bi-degree $p > 0$ can be easily constructed from scalar Green's function ($p = 0$) and the identity double form of bi-degree p. This construction leverages the fact that in flat space there is a path-independent notion of parallel transport. In general, the Green's double forms need to be derived for each bi-degree p separately, though.
We consider Riemannian manifolds with constant curvature. It turns out that Green's double forms can be reduced to certain solutions of the hypergeometric differential equation [2].
The simplest case $n = 2$, $p = 0$ with constant positive curvature has been treated in [3]. The Green's function can be interpreted as electrostatic potential of a point charge on the sphere, with an opposite point charge in the antipodal point. Hodge duality yields the case $n = 2$, $p = 2$. However, an explicit expression of Green's double form for $n = 2$, $p = 1$ seems to be lacking from literature so far.
In this contribution, a closed-form expression for this Green's double form is derived, hence enabling integral operators for magnetostatics on the sphere. Also, the feasibility of the differential forms framework in a non-Euclidean space is highlighted.

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Parallel fast BEM for distributed memory systems

D. Lukáš, P. Kovář, T. Kovářová
TU VSB Ostrava, Czech Republic

We consider Galerkin boundary element method (BEM) accelerated by means of hierarchical matrices (H-matrices) and adaptive cross approximation (ACA). This leads to almost linear complexity O(n log n) of a serial code, where n denotes the number of boundary nodes or elements. Once the setup of an H-matrix is done, parallel assembling is straightforward via a load-balanced distribution of admissible (far-field) and nonadmissible (near-field) parts of the matrix to N concurrent processes. This traditional approach scales the computational complexity as O((n log n) / N). However, the boundary mesh is shared by all processes. We propose a method, which leads to memory scalability $O((n \log n)/sqrtN)$, which is optimal due to the dense matter of BEM matrices. The method relies on our recent results in cyclic decompositions of undirected graphs. Each process is assigned to a subgraph and to related boundary submesh. The parallel scalability of the Laplace single-layer matrix is documented on a distributed memory computer up to 133 cores and three milions of boundary triangles.

Non-symmetric coupling of finite and boundary element methods for the heat equation

M. Neumüller, G. Of, O. Steinbach
TU Graz, Austria

We present some coupling formulations of continuous and discontinuous Galerkin finite element methods with boundary element methods for the heat equation. In particular, we consider the non-symmetric coupling. This enables us to use even discontinuous basis functions on the interface between the subdomains represented by the finite element and boundary lement methods while other formulations require continuity. We will address the error and stability analysis and show the stability and efficiency of the proposed approach for some numerical examples.

Comparison of fast boundary element methods on parametric surfaces

H. Harbrecht, M. Peters
Universität Basel, Switzerland

We compare fast black-box boundary element methods on parametric surfaces in $\mathbb{R}^3$. These are the adaptive cross approximation, the multipole method based on interpolation, and the wavelet Galerkin method.

The special surface representation by piecewise smooth mappings of a single reference domain, i.e. $[0,1]^2 \subset \mathbb{R}^2$, reduces the dimensionality of the underlying problem from 3 to 2. Such surface representations are easily accessible from Computer Aided Design (CAD) and are recently topic of the studies in Isogeometric Analysis. The goal of Isogemetric Analysis is the direct integration of finite element methods into CAD. Thus, we can use the same interface from CAD to parametric surface representations for our purposes.

With the parameterization at hand, we can apply two-dimensional interpolation on the reference domain in the multipole method. A main feature of this approach is that the cluster bases and the respective moment matrices are now independent of the geometry. This results in a superior compression rate of the system matrix compared to other cluster methods.

A surface integral equation domain decomposition method for solving time-harmonic Maxwell Equations in $\mathbb{R}^3$

Zhen Peng, Jin-Fa Lee
The Ohio State University, Columbus, USA

We present a surface integral equation domain decomposition method (SIE-DDM) for time harmonic electromagnetic wave scattering from bounded composite targets. The proposed SIE-DDM starts by partitioning the composite object into homogeneous sub-regions with constant material properties. Each of the sub-regions is comprised of two sub-domains (the interior of the penetrable object, and the exterior free space), separated on the material interface. The interior and the exterior boundary value problems are coupled to each other through the Robin transmission conditions, which are prescribed on the material/domain interface. A generalized combined field integral equation is employed for both the interior and the exterior sub-domains. Convergence studies of the proposed SIE-DDM are included for both single homogeneous objects and composite penetrable objects. Furthermore, a complex large-scale simulation is conducted to demonstrate the capability of the proposed method to model multi-scale electrically large targets.

A new framework for boundary element forward modelling in diffuse optical tomography

M. Schweiger, W. Smigaj, T. Betcke, S. Arridge
University College London, UK

We present a a new software package for boundary element computation in the context of light transport modelling for near-infrared diffuse optical tomography (DOT). The principal application discussed here is brain imaging in clinical diagnostics. Most biological tissues are highly scattering in the infrared wavelength range, and light transport can generally be described as a diffusion process. The layered structure of the tissues of the head, consisting of skin, bone, cerebrospinal fluid, grey and white matter, with piecewise constant optical coefficients, make this problem well-suited for a boundary element approach.
We demonstrate the construction of the linear system for this multilayered BEM problem using surface meshes for the different region boundaries extracted from a segmented head model. The layer hierarchy is encoded in a block matrix structure that is efficiently encoded using the abstract linear operator interface provided by the new BEM++ package. The system is solved efficiently with a low-rank approximation using an H-matrix approach. Examples for the construction of the blocked linear operators using a C++ and python interface will be presented. We are currently working on a GPU-accelerated implementation of the solver, for which preliminary results will be shown.

BEM++ – a new boundary-element library

S. Arridge[a], T. Betcke[b], J. Phillips[b], M. Schweiger[a], W. Śmigaj[b,c]
[a]Department of Computer Science, University College London, London, UK
[b]Department of Mathematics, University College London, London, UK
[c]Faculty of Physics, Adam Mickiewicz University, Poland

This talk will be devoted to BEM++, a new open-source C++/Python library for calculations with the boundary-element method (BEM).
The development of BEM++ started in October 2011 and takes place primarily at the University College London, in cooperation with researchers from the University of Reading and the University of Durham. Its aim is to produce a high-performance, versatile boundary-element library available free of charge for both academic and commercial purposes.
The present version of BEM++ makes it possible to solve boundary-integral problems involving the three-dimensional Laplace and Helmholtz equations. Numerical integration is used to evaluate Galerkin discretisations of integral operators defined on surfaces represented with triangular meshes, managed by means of the Dune library. An interface to the well-established AHMED library gives access to accelerated matrix assembly via the adaptive cross approximation (ACA) algorithm. Parallel matrix assembly on shared-memory machines is also supported. A major objective for the next stage of BEM++ development is a GPU implementation of ACA.
Objects representing discretised boundary-integral operators implement the abstract linear-operator interface defined by the Thyra module from the Trilinos library. As a result, users of BEM++ have direct access to the wide range of iterative solvers provided by Trilinos. We also expect implementation of coupled FEM-BEM methods to be facilitated by the conformance of the discrete boundary-operator classes provided by BEM++ to the standard Thyra interface.
A distinguishing feature of BEM++ is its Python interface. Although the core of the library is written in C++, the Python wrappers allow users to rapidly develop BEM codes and easily visualise and postprocess results of their calculations. The goal of scriptability affects the overall design of BEM++, favouring dynamic rather than static polymorphism.
At the conference, we will give an overview of the design and current functionality of BEM++ and discuss plans for its further development.

Well-conditioned second kind single trace BEM for acoustic scattering at composite objects

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We consider acoustic scattering at composite objects with Lipschitz boundary. The classical first kind approach is ill-conditioned and no preconditioner is available. In contrast to this, a new intrinsically well-conditioned second kind boundary element formulation has been discovered by one of the authors [1]. We adopt this idea and extend it by lifting the formulation from the trace spaces $H^{1/2}(\Gamma) \times H^{-1/2}(\Gamma)$ into the space $L^2(\Gamma) \times L^2(\Gamma)$. This enables us to solely work with discontinuous ansatz functions in order to approximate the unknown boundary data. Experiments show competitive accuracy of the new approach, bounded condition numbers of the Galerkin matrices and fast convergence of GMRES. Several tests also indicate the absence of spurious modes in our new formulation.
In the talk we are going to give an overview over this new method for 2D problems, explain the numerical treatment of the regularized integral kernels and show some numerical results obtained using piecewise constant ansatz functions for the traces.

[1] X. Claeys. A single trace integral formulation of the second kind for acoustic scattering. Technical Report, No. 14, 2011.

[2] E. Spindler. Second Kind Single Trace Boundary Element Methods. Master Thesis, 2012, http://www.sam.math.ethz.ch/ hiptmair/StudentProjects/Spindler.Elke/thesis.pdf.

Boundary integral equations for Helmholtz boundary value and transmission problems

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In this talk we review the formulation of boundary integral equations for the solution of boundary value and transmission problems of the Helmholtz equation. In addition to classical combined formulations for exterior boundary value problems such as Brakhage–Werner and Burton–Miller we also discuss modified combined boundary integral equations which are considered in the natural energy spaces.
For the solution of transmission problems we describe related approaches which are stable for all wavenumbers. While the standard approach relies on the use of modified Robin type transmission conditions, we also discuss other approaches which are based on appropriate combinations of boundary integral equations.

Boundary elements and a smoothed Nash-Hörmander iteration for the Molodensky problem

E. P. Stephan, A. Costea, H. Gimperlein
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We investigate the mathematically justified numerical approximation of the nonlinear Molodensky problem, which reconstructs the surface of the earth from the gravitational potential and the gravity vector. The method, based on a smoothed Nash–Hörmander iteration, solves a sequence of exterior oblique Robin problems and uses a regularization based on a higherorder heat equation to overcome the loss of derivatives in the surface update. In particular, we obtain a quantitative a priori estimate for the error after k steps, justify the use of smoothing operators based on the heat equation, and study the accurate evaluation of the Hessian of the gravitational potential on the surface, using a representation in terms of a hypersingular integral. A boundary element method is used to solve the exterior problem. Numerical results compare the error between the approximation and the exact solution in a model problem.

Solving the Lamé-Navier equations using the convolution quadrature method and the directional fast multipole method

T. Traub
TU Graz, Austria

For the temporal discretization of time domain boundary integral formulations we use the Convolution Quadrature Method. To implement it we employ the approach proposed by Banjai and Sauter where the convolution breaks up into a system of decoupled problems in Laplace domain. The advantage is that these problems are elliptic and, hence, well studied fast solution techniques are applicable.
While the efficiency of $\mathcal{H}$-matrix techniques decreases as the integral kernel becomes more and more oscillatory, Fast Multipole Methods overcome this bottleneck by introducing suitable kernel expansions. A directional approach for oscillatory kernel functions was developed by Engquist et al. Based thereupon a directional Fast Multipole Method (DFMM) was introduced by Messner et al., which we extend to the Lamé kernel. In the case of elastodyanics, we are dealing with two distinct waves, the compression and shear wave. As a consequence, we need to split the fundamental solution, into two oscillatory parts. We can then apply the DFFM technique for each part separately, resulting in a method that does not depend on the oscillatory property of the kernel and scales like $\mathcal{O}(M \log M)$.

Coupled FE-BE eigenvalue problems for fluid-structure interaction

A. Kimeswenger, O. Steinbach, G. Unger
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In this talk we present a coupled finite and boundary element eigenvalue problem formulation for the simulation of the vibro-acoustic behavior of elastic bodies submerged in unbounded fluid domains as submarines in the sea. Usually the fluid is assumed to be incompressible and hence it is modeled by the Laplace equation. In contrast, we do not neglect the compressibility of the fluid but model it by the Helmholtz equation. The resulting coupled eigenvalue problem for the fluid-structure interaction is then nonlinear since the frequency parameter appears nonlinearly in the boundary integral formulation of the Helmholtz equation. We analyze this eigenvalue problem and its discretization in the framework of eigenvalue problems for holomorphic Fredholm operator-valued functions. For the numerical solution of the discretized eigenvalue problem we use the contour integral method which reduces the algebraic nonlinear eigenvalue problem to a linear one. The method is based on a contour integral representation of the resolvent operator and it is suitable for the extraction of all eigenvalues which are enclosed by a given contour. The dimension of the resulting linear eigenvalue problem corresponds to the number of eigenvalues inside the contour. The main computational effort consists in the evaluation of the resolvent operator for the contour integral which requires the solution of several linear systems involving finite and boundary element matrices.

Semi-analytic integration for the hypersingular operator related to the Helmholtz equation in 3D

J. Zapletal

TU Graz, Austria and TU VSB Ostrava, Czech Republic

In the talk we present the application of the boundary element method for solving the Helmholtz equation in 3D. Contrary to the finite element method, one does not need to discretize the whole domain and thus the problem dimension is reduced. This advantage is most pronounced when solving an exterior problem, i.e., a problem on an unbounded domain. A solution to such a problem (the scattering problem) is presented among others. We concentrate on the Galerkin approach known, e.g., from the finite element method and present a combination of analytic and numerical computation of matrices generated by boundary integral operators.

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2012/8 M. Bulicek, P. Pustejovska: On existence analysis of steady flows of generalized Newtonian fluids with concentration dependent power-law index.