

**Technische Universität Graz**



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16. Workshop on  
**Fast Boundary Element Methods in  
Industrial Applications**

Söllerauhaus, 4.–7.10.2018

U. Langer, M. Schanz, O. Steinbach, W. L. Wendland (eds.)

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**Berichte aus dem  
Institut für Angewandte Mathematik**



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## Berichte aus dem Institut für Angewandte Mathematik

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Technische Universität Graz  
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## Program

Thursday, October 4, 2018	
15.00	Coffee
16.30–16.45	Opening
16.45–17.15	C. Jerez–Hanckes (Santiago de Chile) High–order Galerkin method for Helmholtz and Laplace problems on multiple open arcs
17.15–17.45	S. Dohr (Graz) Distributed space–time BEM for parabolic problems
17.45–18.15	J. Hauser (Graz) Space–time variational formulations for Maxwell’s equations
18.30	Dinner
Friday, October 5, 2018	
8.00–9.00	Breakfast
9.00–9.30	L. Kielhorn (Tailsit, Graz) No solutions. Just problems.
9.30–10.00	M. Taus (MIT/Graz) A scalable parallel preconditioner for the high–frequency Helmholtz equation
10.00–10.30	G. Unger (Graz) Boundary element methods for plasmonic resonance problems
10.30–11.00	Coffee
11.00–11.30	F. Kpadonou (Palaiseau) Efficient preconditioners for the $\mathcal{H}$ –matrix based iterative solver for 3D oscillatory kernels
11.30–12.00	A. Kleanthous (London) Preconditioning for electromagnetic scattering of multiple absorbing dielectric objects
12.00–12.30	C. Urzua–Torres (Graz) Operator preconditioning for the electric field integral equation on screens
12.30	Lunch
14.30–15.00	T. Betcke (London) Strong forms of Galerkin discretisations and their applications to integral equations
15.00–15.30	J. Dölz (Darmstadt) On the best approximation of the hierarchical matrix product
15.30–16.00	Coffee
16.00–16.30	L. Giacomel (Zürich) BEM on complex screens
16.30–17.00	D. Seibel (Saarbrücken) The Virtual Element Method for elliptic partial differential equations on polygonal and polyhedral meshes
17.00–17.15	Break
17.15–17.45	J. Stocck (Edinburgh) Space–time adaptive FEM for nonlocal parabolic variational inequalities
17.45–18.15	M. Scroogs (London) Weak imposition of boundary conditions using a penalty method
18.30	Dinner

Saturday, October 6, 2018	
8.00–9.00	Breakfast
9.00–9.30	H. Gimperlein (Edinburgh) Stabilized boundary elements for dynamic contact problems for the wave equation
9.30–10.00	D. Pölz (Graz) A space–time collocation scheme for retarded potential integral equations
10.00–10.30	M. Zank (Graz) Inf–sup stable variational formulations for the wave equation
10.30–11.00	Coffee
11.00–11.30	C. Özdemir (Hannover) A toolbox for higher–order time domain boundary element method
11.30–12.00	M. Merta (Ostrava) Distributed fast boundary element methods
12.00–12.30	P. Zaspel (Basel) Scalable parallel BEM solvers on many core clusters
12.30	Lunch
13.30–18.00	Hiking Tour
18.30	Dinner
Sunday, October 7, 2018	
8.00–9.00	Breakfast
9.00–9.30	H. Harbrecht (Basel) Multilevel quadrature for elliptic problems on random domains by the coupling of FEM and BEM
9.30–10.00	D. Lukas (Ostrava) A boundary element method for homogenization of periodic structures
10.00–10.30	G. Of (Graz) Some boundary element methods for multiply–connected domains
10.30–11.00	Coffee

## **Strong forms of Galerkin discretisations and their applications to integral equations**

Timo Betcke

University College London, UK

In this talk we revisit generalised Riesz maps (or mass matrices) in the context of integral operators. We consider strong forms as the product of an inverse mass matrix and a Galerkin discretisation of an integral operator. Based on this notion we define a discrete operator product algebra for Galerkin discretisations and its implementation in software. Several examples from acoustics and Maxwell problems will demonstrate how this algebra makes it very simple to describe in software complex operator preconditioning modalities.

## Distributed space-time BEM for parabolic problems

Stefan Dohr  
TU Graz, Austria

Space-time discretization methods became very popular in recent years due to their ability to drive adaptivity in space and time simultaneously, and to use parallel iterative solution strategies for time-dependent problems. However, in order to solve the global space-time system the application of an efficient parallelization technique is required.

In this talk we consider the heat equation as a model problem and introduce a parallel solver for the discretized boundary integral equations, for both Dirichlet and Neumann boundary conditions. The space-time boundary mesh is decomposed into a given number of submeshes. Pairs of the submeshes represent blocks in the system matrices. Due to the structure of the matrices one has to design a suitable scheme for the distribution of the matrix blocks among compute nodes in order to get an efficient method. In our case the distribution is based on a cyclic decomposition of complete graphs. The results can be transferred to parabolic transmission problems as well. We present numerical tests to evaluate the efficiency of the proposed parallelization approach.

The presented parallel solver is based on joint work with G. Of from TU Graz, J. Zapletal and M. Merta from the Technical University of Ostrava.

## On the best approximation of the hierarchical matrix product

Jürgen Dölz<sup>1</sup>, Helmut Harbrecht<sup>2</sup>, Michael D. Multerer<sup>2</sup>

<sup>1</sup>TU Darmstadt, Germany, <sup>2</sup>Universität Basel, Switzerland

We consider the computation of a hierarchical matrix approximation to the product of pseudo-differential operators. In the discrete setting, this approximation can be seen as the product of two hierarchical matrices. Although the classical arithmetic of hierarchical matrices can compute this approximation in almost linear time, its efficiency of the algorithm is based on a recursive scheme, which makes the error analysis quite involved. Therefore, we discuss a new algorithmic framework for the multiplication of hierarchical matrices. It improves currently known implementations by reducing the multiplication of hierarchical matrices towards finding a suitable low-rank approximation of sums of matrix-products. We propose several compression schemes to address this task. As a consequence, we are able to compute the best-approximation of hierarchical matrix products.

## BEM on complex screens

Lorenzo Giacomel<sup>1</sup>, R. Hiptmair<sup>1</sup>, C. Urzua–Torres<sup>2</sup>

<sup>1</sup>ETH Zürich, Switzerland, <sup>2</sup>TU Graz, Austria

First-kind boundary integral equations for solving scalar elliptic boundary value problems in the exterior of an orientable Lipschitz screen  $\Gamma \subset \mathbb{R}^d$  are well established. They are set in trace spaces  $\tilde{H}^{\frac{1}{2}}(\Gamma)$  and  $\tilde{H}^{-\frac{1}{2}}(\Gamma)$ , respectively, and their unknowns are the *jumps* of the fields across  $\Gamma$ .

Let us now consider multi-screens,  $d - 1$ -dimensional varieties composed of a few orientable Lipschitz parts attached to each other at junctions, where several of them can meet. What is the meaning of a jump of a function at these junctions? This issue can be resolved by switching to a quotient space view of traces. This naturally leads to the notion of multi-trace spaces of multi-valued functions, encompassing single-valued “single-trace” functions. Jump spaces emerge as the quotient spaces of the two.

Our key idea is to switch to the quotient-space approach also for the sake of Galerkin discretization. Instead of trying to figure out boundary element (BE) subspaces of jump traces, we start from the straightforward conforming BE discretization of multi-trace spaces. On them the boundary integral equations (BIEs) will fail to have a unique solution, but are still consistent, therefore amenable to being solved by a conjugate gradient iterative solver. Thus, we implicitly recover the BIE on the quotient space. We also give numerical demonstrations for the feasibility of this approach.

## **Stabilized boundary elements for dynamic contact problems for the wave equation**

H. Gimperlein, G. Barrenechea, C. Ozdemir, J. Stoeck  
Heriot-Watt University Edinburgh, UK

We discuss the numerical analysis of the Signorini problem for the time dependent wave equation. The equation is reduced to a variational inequality for the Poincare-Steklov operator on the contact boundary, which numerically is realized in terms of retarded potentials. We review the existence of solutions to the dynamic contact problem and, assuming existence, discuss the a priori analysis of an equivalent mixed formulation, as well as its stabilization based on local projections. Even for time independent problems our approach provides a simplified stabilization of boundary element discretizations.

Numerical results illustrate the efficiency of our methods for dynamic contact in three dimensions.

# **High-order Galerkin method for Helmholtz and Laplace problems on multiple open arcs**

Carlos Jerez–Hanckes

Pontificia Universidad Catolica de Chile

We present a spectral numerical scheme for solving Helmholtz and Laplace problems with Dirichlet boundary conditions on a finite collection of open arcs in  $\mathbb{R}^2$ . An indirect boundary integral method is employed, giving rise to a first kind formulation whose variational form is discretized using weighted Chebyshev polynomials. Well-posedness of both continuous and discrete problems is established as well as spectral convergence rates under the existence of analytic maps to describe the arcs. In order to reduce computation times, a simple matrix compression technique based on sparse kernel approximations is developed. Numerical results provided validate our claims.

## **Multilevel quadrature for elliptic problems on random domains by the coupling of FEM and BEM**

Helmut Harbrecht

Universität Basel, Switzerland

Elliptic boundary value problems which are posed on a random domain can be mapped to a fixed, nominal domain. The randomness is thus transferred to the diffusion matrix and the loading. This domain mapping method is quite efficient for theory and practice, since only a single domain discretization is needed. Nonetheless, it is not useful for applying multilevel accelerated methods to efficiently deal with the random parameter. This issues from the fact that the domain discretization needs to be fine enough in order to avoid indefinite diffusion matrices. To overcome this obstruction, we are going to couple the finite element method with the boundary element method. In this talk, we verify the required regularity with respect to the random perturbation field, derive the coupling formulation, and show by numerical results that the approach is feasible.

## Space-time variational formulations for Maxwell's equations

Julia Hauser, Olaf Steinbach  
TU Graz, Austria

Maxwell's equations are everywhere in electro magnetic problems. There are many approaches to solve these equations. Our approach is to consider time as another dimension and look at Maxwell's equations in a corresponding 4D space-time setting. To be able to solve boundary integral equations in this setting, we first need to take a look at variational formulations in the domain and their corresponding finite element methods. For this purpose we consider the equations on a bounded Lipschitz domain in space and a bounded interval in time. The electric permittivity and magnetic permeability shall be symmetric, positive definite and bounded matrix functions.

## **No solutions. Just problems.**

L. Kielhorn, T. Rüberg, J. Zechner  
TAILSIT GmbH, Graz, Austria

This talk is concerned with open problems related to computational electromagnetism. TAILSIT gained quite some expertise in the development and implementation of FEM-BEM coupling schemes for electromagnetic phenomena such as magnetostatics and eddy-currents. While these coupling schemes are based on a sound mathematical foundation, in practice however, one faces some issues which are not covered by the theory. These issues are either related to the the coupling scheme itself, or—in many instances—to the underlying Boundary Element Method. Thus, some of the problems this talk addresses are of general nature rather than problem-specific. The following list is intended to give a rough overview on some of the issues that will be discussed:

- The one equation coupling is stable but reveals severe accuracy problems.
- Our preconditioner for the coupled system of equations seems to be not optimal w.r.t the material parameter.
- The ACA fails for magnetostatics and eddy-currents (*... but FMM succeeds*).
- Implementing symmetry constraints poses problems—especially in conjunction with cohomology vector fields.
- Distributed computing: Domain decomposition vs. parallel FMM—what is the best strategy?

The above topics will be covered in more detail within the talk. We hope this presentation will create some curiosity out of which solutions might emerge.

## Preconditioning for electromagnetic scattering of multiple absorbing dielectric objects

A. Kleanthous<sup>1</sup>, T. Betcke<sup>1</sup>, D. Hewett<sup>1</sup>, M. Scroggs<sup>1</sup>, and A. J. Baran<sup>2,3</sup>

<sup>1</sup>University College London, UK,

<sup>2</sup>Met Office, Exeter, UK,

<sup>3</sup>University of Hertfordshire, UK

In recent years Calderón preconditioning [1] and appropriate use of basis functions [2] have become a popular strategy to speed up the iterative solution of electromagnetic scattering by a single dielectric particle. In this talk we will discuss a different type of preconditioning for single scattering problems, namely mass-matrix preconditioning, and then extend the ideas of Calderón and mass-matrix preconditioning in the case of scattering by multiple absorbing dielectric objects [3]. We will compare the methods and then demonstrate applications of the above preconditioners in the area of light scattering by single and multiple complex ice crystals found in cirrus clouds, using the boundary element library Bempp [4].

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# Efficient preconditioners for the $\mathcal{H}$ -matrix based iterative solver for 3D oscillatory kernels

F. D. Kpadonou, S. Chaillat, P. Ciarlet

Université Paris-Saclay ENSTA-UMA, Palaiseau, France

We are concerned in this contribution with the improvement of the efficiency of Boundary Element Methods (BEMs) for 3D frequency domain oscillatory problems. The need of efficient tools is crucial for the simulation of many real-life problems such as soil-structure interaction, site-effects phenomenon, non-destructive controle of structure (e.g. in nuclear area), and for the modelling and design of anti-noise walls.

On the one hand, BEMs are based on the discretization of boundary integral equations [2] such that only the domain boundary is meshed. On the other hand, they lead to a linear system with a fully-populated influence matrix, conversely to standard volume methods such as finite elements. Hence standard BEM solvers lead to high computational costs both in terms of time and memory requirements. This drawback prevents to treat large scale three-dimensional problems. Over the last decades, various solutions have been proposed in order to circumvent the full assembly and storage of the matrix. The most popular are probably the  $\mathcal{H}$ -matrix technique [1] and the Fast Multipole Method [4] to compute the integral operators, i.e the matrix-vector product which is, indeed, the essential operation for an iterative solver.

Since the matrix-vector product is log-linear factor with these methods, iterative solvers are very appealing. However the definition of an efficient preconditioner or the number of iterations is still an issue and a limiting factor to treat large scale problems. In the typical case of FMM based iterative solver, only the near-field contributions of the system matrix are available. Therefore, one is rapidly limited in the possibilities for the setting of that preconditioner (SPAI, LU factorization, etc..). In the case of  $\mathcal{H}$ -matrix, although two kinds of storage are used, namely the full storage for non-admissible blocks and the low-rank representation for the admissible ones, the system matrix is available. We investigate the definition of an efficient preconditioner for the  $\mathcal{H}$ -matrix based solver [3]. We consider the General Minimal Residual (GMRES) [5,6] based algorithms for the iterative solver. Several numeral tests are shown to illustrate the efficiency of the different applicant preconditioner tested.

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## A boundary element method for homogenization of periodic structures

D. Lukáš<sup>1</sup>, J. Bouchala<sup>1</sup>, J. Zapletal<sup>1</sup>, and G. Of<sup>2</sup>

<sup>1</sup>TU VSB Ostrava, Czech Republic, <sup>2</sup>TU Graz, Austria

Solution to a boundary value problem involving materials with composite microstructure is computationally demanding. Therefore, we look for homogeneous (constant) material coefficients imitating the original microstructure so that the solution to the original problem with a highly oscillating material function is in a sense close to the solution of a related problem with the constant material function.

In case of periodic structures the homogenized coefficients are calculated via an auxiliary partial differential equation in the periodic cell. Typically a volume finite element discretization is employed for the numerical solution. In this talk we reformulate the problem as a boundary integral equation using Steklov-Poincare operators. The resulting boundary element method introduces discretization along the boundary of the periodic cell and the interface between the materials within the cell. We prove that the homogenized coefficients converge super-linearly with the mesh size. The proof relies on the inf-sup stability of the Steklov-Poincare operator defined on a multiply-connected domain. To our best knowledge this case has not been treated in literature yet. We support the theory with examples in 2 and 3 dimensions.

## Distributed fast boundary element methods

Michal Merta, Jan Zapletal  
TU VSB Ostrava, Czech Republic

We present an approach for a distributed memory parallelization of the boundary element method. The input mesh is decomposed into submeshes and the respective matrix blocks are distributed among computational nodes (MPI processes). The distribution which takes care of the load balancing during the system matrix assembly and matrix-vector multiplication is based on the cyclic graph decomposition. Moreover, since the individual matrix blocks are approximated using the adaptive cross approximation method, we describe its modification capable of dealing with zero blocks in the double layer operator matrix since these are usually problematic when using the original ACA algorithm. Convergence and parallel scalability of the method are demonstrated on the half- and full-space sound scattering problems modeled by the Helmholtz equation.

## **A toolbox for higher-order time domain boundary element method**

Ceyhun Özdemir

Leibniz Universität Hannover, Germany

We discuss aspects of the theory and implementation of higher-order h and p-versions of the time domain boundary element method for the wave equation in  $\mathbb{R}^3$ . Numerical examples illustrate the convergence properties and relevance to real-world problems from traffic noise. We particularly address the performance of these methods for problems in polyhedral domains or outside a screen, where the solution exhibits singularities at the edges and corners. For the h-version, graded meshes are shown to recover optimal approximation rates for the solution to both Dirichlet and Neumann problems. Adaptive mesh refinement procedures based on a reliable and efficient a posteriori error estimate are shown to recover the convergence rates known for time-independent problems. In the second part of the talk we introduce a p-version of the time domain boundary element method, discuss its conforming implementation and the practical solution of the resulting space-time systems.

## Some boundary element methods for multiply-connected domains

Dalibor Lukas<sup>1</sup>, Günther Of<sup>2</sup>, Olaf Steinbach<sup>2</sup>

<sup>1</sup>TU VSB Ostrava, Czech Republic, <sup>2</sup>TU Graz, Austria

In case of multiply-connected domains, some properties of the boundary integral operators differ from the setting of a simply-connected domain. In particular, the kernel and the ellipticity property of the hypersingular operator are different. As a consequence, some boundary integral formulations, like the symmetric formulation of mixed boundary value problems, may have a larger kernel than the considered problem itself.

We will discuss some details of the changes in the analysis of the boundary integral operators and of the considered boundary integral formulations. We will show some examples of failures of specific formulations and how to fix these by appropriate modifications.

## A space-time collocation scheme for retarded potential integral equations

Dominik Pölz, Martin Schanz

TU Graz, Austria

We discuss a boundary element method for the wave equation in 3D based on retarded potential integral equations. Most existing approximation methods for such integral equations discretize space and time separately, however, our goal is to develop a discretization scheme that does not rely on this separation. The key idea of such space-time methods is to treat the time variable as a coordinate similar to the spatial ones. This enables the application of well-established finite and boundary element technology for stationary problems to time domain boundary integral equations.

To achieve such a discretization, the lateral boundary of the space-time cylinder is decomposed into an unstructured tetrahedral mesh. On this mesh, standard finite element spaces are employed to approximate the surface densities. The central challenge is the evaluation of the retarded layer potentials, which integrate over the intersection of the space-time mesh and the boundary of the backward light cone. Since these integrals can be quite complicated we restrict our considerations to collocation methods only. For tetrahedral space-time meshes we provide an accurate numerical integration scheme for the pointwise evaluation of retarded layer potentials. Several numerical examples illustrate the capacity of the method.

The talk concludes by addressing critical issues encountered in this early stage of development as well as the limitations of the presented approach.

## Weak imposition of boundary conditions using a penalty method

Matthew Scroggs, Timo Betcke, Erik Burman  
University College London, UK

In recent years, Nitsche's method has become increasingly popular within the finite element community as a method for weakly imposing boundary conditions. Inspired by this, we propose a penalty-based method for weakly imposing boundary conditions within boundary element methods.

We consider boundary element methods where the Calderon projector is used for the system matrix and boundary conditions are weakly imposed using a particular variational boundary operator. Regardless of the boundary conditions, both the primal trace variable and the flux are approximated. We focus on the imposition of Dirichlet, mixed Dirichlet, Neumann, and Robin conditions for Laplace problems. The theory is illustrated by a series of numerical examples using the software library Bempp.

## **The Virtual Element Method for elliptic partial differential equations on polygonal and polyhedral meshes**

Daniel Seibel

Universität des Saarlandes, Saarbrücken, Germany

The virtual element method (VEM) is a generalisation of the Finite Element Method (FEM) to support polygonal and polyhedral meshes. In contrast to classical triangulations, polygonal/polyhedral meshes consist of almost arbitrary shaped elements and are specifically suited for adaptive refinement and coarsening, since post-processing to maintain the mesh admissibility is rendered unnecessary. Similar to other generalised FEM, the shape functions used in the VEM, called virtual element functions, are defined as solutions to local boundary value problems on the elements of the mesh. The virtual element space, which is spanned by these functions, is designed to contain some polynomial subspace, and other, possibly non-polynomial, virtual functions, which are only given implicitly. The key feature here is that the virtual functions are not evaluated at any time, but all computations resolve around the polynomial component of the space. In this talk, we give an overview of the VEM in 2D and present numerical examples afterwards.

## Space-time adaptive FEM for nonlocal parabolic variational inequalities

Jacob Stoeck

Heriot-Watt University Edinburgh, UK

We present an a priori and a posteriori error analysis for finite element discretizations of elliptic and parabolic variational inequalities involving integral operators of order  $s \in (0, 2)$ . A typical example is given by the fractional Laplacian in a bounded Lipschitz domain, and we consider variational and mixed formulations of general dynamic contact problems involving non-penetration or friction. The a posteriori error estimate leads to space-time adaptive mesh refinement procedures. Numerical experiments confirm the theoretical results and show the advantages and limitations of the adaptive methods.

# A scalable parallel preconditioner for the high-frequency Helmholtz equation

Matthias Taus

Massachusetts Institute of Technology, USA

In many science and engineering applications, solving time-harmonic high-frequency wave propagation problems quickly and accurately is of paramount importance. For example, in geophysics, particularly in oil exploration, such problems can be the forward problem in an iterative process for solving the inverse problem of subsurface inversion. It is important to solve these wave propagation problems accurately in order to efficiently obtain meaningful solutions of the inverse problems: low order forward modeling can hinder convergence. Additionally, due to the volume of data and the iterative nature of most optimization algorithms, the forward problem must be solved many times. Therefore, a fast solver is necessary to make solving the inverse problem feasible. For time-harmonic high-frequency wave propagation, obtaining both speed and accuracy is historically challenging.

Recently, there have been many advances in the development of fast solvers for such problems, including methods which have linear complexity with respect to the number of degrees of freedom. While most methods scale optimally only in the context of low-order discretizations and smooth wave speed distributions, the method of polarized traces has been shown to retain optimal scaling for high-order discretizations, such as hybridizable discontinuous Galerkin methods and for highly heterogeneous (and even discontinuous) wave speeds. The resulting fast and accurate solver is consequently highly attractive for geophysical applications. To date, this method relies on a layered domain decomposition together with a preconditioner applied in a sweeping fashion, which has limited straight-forward parallelization.

In this work, we introduce a new version of the method of polarized traces which reveals more parallel structure than previous versions while preserving all of its other advantages. We achieve this by further decomposing each layer and applying the preconditioner to these new components separately and in parallel. We demonstrate that this produces an even more effective and parallelizable preconditioner for a single right-hand side. As before, additional speed can be gained by pipelining several right-hand-sides.

## Boundary element methods for plasmonic resonance problems

U. Hohenester<sup>1</sup>, A. Trügler<sup>1</sup>, G. Unger<sup>2</sup>

<sup>1</sup>KFU Graz, Austria, <sup>2</sup>TU Graz, Austria

The concept of resonances and modes for the description of particle plasmons has recently received great interest, both in the context of efficient simulations as well as for an intuitive interpretation in physical terms. While resonance modes have been successfully employed for geometries whose optical response is governed by a few modes only, the resonance mode description exhibits considerable difficulties for larger nanoparticles with their richer mode spectra. In this talk we analyze the problem using a boundary element method approach and identify the fixed link between the electric and magnetic components in the modal expansion of the optical response as the main source for this shortcoming. We suggest a novel modal approximation scheme that allows in principle to overcome this problem by proposing separate coefficients of the the electric and magnetic components of the modal expansion.

## Operator preconditioning for the electric field integral equation on screens

Ralf Hiptmair<sup>1</sup>, Carolina Urzúa-Torres<sup>2</sup>

<sup>1</sup>ETH Zürich, Switzerland, <sup>2</sup>TU Graz, Austria

We consider the electric field integral equation (EFIE) arising from the scattering of time-harmonic electromagnetic waves by a perfectly conducting screen. When discretizing the EFIE by means of low-order Galerkin boundary element methods (BEM), one obtains linear systems that are ill-conditioned on fine meshes. This makes iterative solvers perform poorly and motivates the use of preconditioning. The construction of a suitable preconditioner for the EFIE on screens poses some challenges one should take into account. On the one hand, the energy trace spaces involved are different from the closed surface setting. On the other hand, since in this case the solution features edge singularities, it is important that the preconditioner can also handle meshes refined towards the boundary of the screen. For these two reasons, the standard “Calderón preconditioning” technique is suboptimal when dealing with screens [1].

In this talk, we present a new strategy to build a preconditioner for the EFIE on screens. First, we find a compact equivalent inverse of the EFIE operator on the disk using recently found Calderón-type identities [2]. Then, we use this to construct an operator preconditioner on more general screens. This approach not only offers  $h$ -independent condition numbers, but it allows for non-uniform meshes without additional computational effort. We provide some numerical results to verify our theoretical findings.

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## Inf–sup stable variational formulations for the wave equation

Olaf Steinbach, Marco Zank

TU Graz

For the discretisation of time-dependent partial differential equations usually explicit or implicit time stepping schemes are used. An alternative approach is the usage of space-time methods, where the space-time domain is discretised and the resulting global linear system is solved at once. In this talk the model problem is the scalar wave equation. First, a brief overview of known results for the wave equation and its boundary integral equations is presented. Second, two space-time approaches for the second order wave equation are introduced. Uniqueness and existence including corresponding inf-sup conditions are proven. In both cases the starting point is a Hilbert space  $H$  and then the idea is to use a completion procedure to define a subspace  $H_0 \subset H$  where a Poincaré-Friedrichs type inequality holds. This idea leads to a uniquely solvable variational formulation in  $H_0$ . In the first approach the second order wave equation is considered in the sense of  $L^2$ , whereas in the second one a weaker sense than  $L^2$  is examined. Finally, examples for solutions of the wave equation are presented and discussed.

## Scalable parallel BEM solvers on many-core clusters

Peter Zaspel

Universität Basel, Switzerland

Our aim is to solve large scale problems discretized by the boundary element method. To this end, we propose to use parallel computers equipped with graphics processing units to assemble and solve the linear systems involved in the discretization. Depending on the application case, we either assemble the full dense system matrix (in parallel) or we compress the matrix by hierarchical matrices with adaptive cross approximation. In either case, Krylov subspace solvers are applied to solve the linear system. Our multi-GPU parallel implementation is achieved by porting a sequential CPU BEM code to GPUs and by parallelizing our GPU-based hierarchical matrix library (hmglib). In our presentation, we will give details on the parallel implementation and we will show our latest parallel performance benchmarks.

## Participants

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- 2014/1 K. Bandara, F. Cirak, G. Of, O. Steinbach, J. Zapletal: Boundary element based multiresolution shape optimisation in electrostatics.
- 2014/2 T. X. Phan, O. Steinbach: Boundary integral equations for optimal control problems with partial Dirichlet control.
- 2014/3 M. Neumüller, O. Steinbach: An energy space finite element approach for distributed control problems.
- 2014/4 L. John, O. Steinbach: Schur complement preconditioners for boundary control problems.
- 2014/5 O. Steinbach: Partielle Differentialgleichungen und Numerik.
- 2014/6 T. Apel, O. Steinbach, M. Winkler: Error estimates for Neumann boundary control problems with energy regularization.
- 2014/7 G. Haase, G. Plank, O. Steinbach (eds.): Modelling and Simulation in Biomechanics. Book of Abstracts.