

Julia Ines Mareike Hauser, BSc

The inverse problem for the dielectric conductivity

MASTER'S THESIS

to achieve the university degree of

Diplom-Ingenieur

Masterstudium Technomathematik

submitted to

Graz University of Technology

Supervisor

Prof. Dr. O. Steinbach

Institute of Computational Mathematics

Graz, Oktober 2017

Contents

Posing the problem	7
1. Introduction	9
1.1. Solvability of the original problem	9
1.2. Definition of the Dirichlet-to-Neumann Operator	10
1.3. Transformation to the Schrödinger equation	12
1.4. Definition of the DtN operator for the Schrödinger equation	15
2. Uniqueness of the inverse problem	19
2.1. Solution of Schrödinger equation	20
2.2. Uniqueness of the inverse problem	27
3. An integral equation to compute $\tilde{S}_q\psi_\eta$	31
3.1. A formula for ψ_η on Γ	31
3.2. An integral equation for $\tilde{S}_q\psi_\eta$	37
3.2.1. Solvability	38
4. Solution of the inverse problem	41
4.1. Computation of q	41
4.1.1. Computation of q with the Fourier transform	42
4.1.2. Computation of q with G_0	43
4.1.3. Computation of q with basis functions	44
4.2. Computation of σ	44
4.3. The algorithm	45
4.3.1. Compute the Neumann data	45
4.3.2. Compute q	46
4.3.3. Compute σ	46
4.3.4. Summary	46
5. The reconstruction problem	49
5.1. Introduction	49
5.2. Discretization	50
5.2.1. Compute q with the Fourier transform	52
5.2.2. Compute q with G_0	54
5.2.3. Second way to compute q with G_0	57
5.2.4. Compute q with basis functions	59

6. Numerical results	61
6.1. Approximation of q	61
6.1.1. Results for approximating q with the Fourier transform	61
6.1.2. Results for approximating q with G_0	63
6.1.3. Results for the second way to approximate q with G_0	63
6.1.4. Results for approximating q with basis functions	66
6.2. Approximation of σ	69
7. Conclusion	71
Appendices	73
A. Definitions	75
B. Tools	81
C. The space V_ξ	87
D. Comparison between different $\tilde{\Omega}$	89
D.1. With refinement steps beforehand	92
Bibliography	95